Chapter 1

Differentiation

Exercise Set 1.1

1. We solve the equation:
$-3x = 6$
$x = -2$
Therefore, As x approaches -2, the value of $-3x$ approaches 6.

2. As x approaches 7, the value of $x-2$ approaches 5.

3. The notation $\lim_{x\to 4} f(x)$ is read "the limit, as x approaches 4, of $f(x)$."

4. The notation $\lim_{x\to 1} g(x)$ is read "the limit, as x approaches 1, of $g(x)$."

5. The notation $\lim_{x\to 5^-} F(x)$ is read "the limit, as x approaches 5 from the left, of $F(x)$."

6. The notation $\lim_{x\to 4^+} G(x)$ is read "the limit, as x approaches 4 from the right, of $G(x)$."

7. The notation $\lim_{x\to 2^+}$ is read "the limit, as x approaches 2 from the right."

8. The notation $\lim_{x\to 3^-}$ is read "the limit, as x approaches 3 from the left."

9. The notation $\lim_{x\to 5}$ is read "the limit, as x approaches 5."

10. The notation $\lim_{x\to \frac{1}{2}}$ is read "the limit, as x approaches $\frac{1}{2}$."

11. a) As inputs x approach 3 from the left, outputs $f(x)$ approach 1. That is,
$\lim_{x\to 3^-} f(x) = 1.$

b) As inputs x approach 3 from the right, outputs $f(x)$ approach 2. Thus the limit from the right is 2. That is,
$\lim_{x\to 3^+} f(x) = 2.$

c) From part (a) and part(b) we know that $\lim_{x\to 3^-} f(x) = 1$ and $\lim_{x\to 3^+} f(x) = 2$. Since the limit from the left, 1, is not the same as the limit from the right, 2, $\lim_{x\to 3} f(x)$ *does not exist.*

12. a) As inputs x approach -1 from the left, outputs $f(x)$ approach -3. Thus the limit from the left is -3. That is,
$\lim_{x\to -1^-} f(x) = -3.$

b) As inputs x approach -1 from the right, outputs $f(x)$ approach -3. That is,
$\lim_{x\to -1^+} f(x) = -3.$

c) From part (a) and part (b) we know that $\lim_{x\to -1^-} f(x) = -3$ and $\lim_{x\to -1^+} f(x) = -3.$ Since the limit from the left, $-3,$ is the same as the limit from the right, $\lim_{x\to 3} f(x) = -3.$

13. a) As inputs x approach -2 from the left, outputs $g(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to -2^-} g(x) = 4$.

b) As inputs x approach -2 from the right, outputs $g(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to -2^+} g(x) = 2$.

c) From part (a) and part (b) we know that $\lim_{x\to -2^-} g(x) = 4$ and $\lim_{x\to -2^+} g(x) = 2$. Since the limit from the left, 4, is not the same as the limit from the right, 2, $\lim_{x\to -2} g(x)$ *does not exist.*

14. a) As inputs x approach 4 from the left, outputs $g(x)$ approach -1. Thus the limit from the left is -1. That is, $\lim_{x\to 4^-} g(x) = -1$.

b) As inputs x approach 4 from the right, outputs $g(x)$ approach -1. Thus the limit from the right is -1. That is, $\lim_{x\to 4^+} g(x) = -1$.

c) Since the limit from the left, -1, is the same as the limit from the right, -1, we have $\lim_{x\to 4} g(x) = -1$.

15. As inputs x approach 2 from the left, outputs $F(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to 2^-} F(x) = 4$. As inputs x approach 2 from the right, outputs $F(x)$ approach 4. Thus the limit from the right is 4. That is, $\lim_{x\to 2^+} F(x) = 4$.
Since the limit from the left, 4, is the same as the limit from the right, 4, we have $\lim_{x\to 2} F(x) = 4$.

16. We have, $\lim_{x\to -3^-} F(x) = 5$ and $\lim_{x\to -3^+} F(x) = 5$. Therefore, $\lim_{x\to -3} F(x) = 5$.

17. As inputs x approach -5 from the left, outputs $F(x)$ approach 0. Thus the limit from the left is 0. That is, $\lim_{x\to -5^-} F(x) = 0$.
As inputs x approach -5 from the right, outputs $F(x)$ approach 0. Thus the limit from the right is 0. That is, $\lim_{x\to -5^+} F(x) = 0$.
Since the limit from the left, 0, is the same as the limit from the right, 0, we have $\lim_{x\to -5} F(x) = 0$.

18. We have, $\lim_{x\to -2^-} F(x) = 4$ and $\lim_{x\to -2^+} F(x) = 2$. Therefore, $\lim_{x\to -2} F(x)$ *does not exist.*

19. As inputs x approach 6 from the left, outputs $F(x)$ approach 0. Thus the limit from the left is 0. That is, $\lim_{x\to 6^-} F(x) = 0$.
As inputs x approach 6 from the right, outputs $F(x)$ approach 0. Thus the limit from the right is 0. That is, $\lim_{x\to 6^+} F(x) = 0$.
Since the limit from the left, 0, is the same as the limit from the right, 0, we have $\lim_{x\to 6} F(x) = 0$.

20. We have, $\lim_{x\to 4^-} F(x) = 2$ and $\lim_{x\to 4^+} F(x) = 2$. Therefore, $\lim_{x\to 4} F(x) = 2$.

21. As inputs x approach -2 from the left, outputs $F(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to -2^-} F(x) = 4$.

22. As inputs x approach -2 from the right, outputs $F(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to -2^+} F(x) = 2$.

23. As inputs x approach 0 from the left, outputs $G(x)$ approach 3. Thus the limit from the left is 3. That is, $\lim_{x\to 0^-} G(x) = 3$.
As inputs x approach 0 from the right, outputs $G(x)$ approach 3. Thus the limit from the right is 3. That is $\lim_{x\to 0^+} G(x) = 3$.
Since the limit from the left, 3, is the same as the limit from the right, 3, we have $\lim_{x\to 0} G(x) = 3$.

24. We have $\lim_{x\to -2^-} G(x) = 1$ and $\lim_{x\to -2^+} G(x) = 1$. Therefore, $\lim_{x\to -2} G(x) = 1$.

25. As inputs x approach 1 from the right, outputs $G(x)$ approach -1. Thus the limit from the right is -1. That is, $\lim_{x\to 1^+} G(x) = -1$

26. As inputs x approach 1 from the left, outputs $G(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to 1^-} G(x) = 4$.

27. As inputs x approach 1 from the left, outputs $G(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to 1^-} G(x) = 4$.
The solution is continued on the next page.

As inputs x approach 1 from the right, outputs $G(x)$ approach -1. Thus the limit from the right is -1. That is, $\lim_{x\to 1^+} G(x) = -1$.
Since the limit from the left, 4, is not the same as the limit from the right, -1, we have $\lim_{x\to 1} G(x)$ *does not exist.*

28. As inputs x approach 3 from the left, outputs $G(x)$ approach 0. Thus the limit from the left is 0. That is, $\lim_{x\to 3^-} G(x) = 0$

29. As inputs x approach 3 from the left, outputs $G(x)$ approach 0. Thus the limit from the left is 0. That is, $\lim_{x\to 3^-} G(x) = 0$.
As inputs x approach 3 from the right, outputs $G(x)$ approach 0. Thus the limit from the right is 0. That is, $\lim_{x\to 3^+} G(x) = 0$.
Since the limit from the left, 0, is the same as the limit from the right, 0, we have $\lim_{x\to 3} G(x) = 0$.

30. As inputs x approach 3 from the right, outputs $G(x)$ approach 0. Thus the limit from the right is 0. That is, $\lim_{x\to 3^+} G(x) = 0$.

31. As inputs x approach 2 from the left, outputs $H(x)$ approach 1. Thus the limit from the left is 1. That is, $\lim_{x\to -2^-} H(x) = 1$

32. We have $\lim_{x\to -3^-} H(x) = 0$ and $\lim_{x\to -3^+} H(x) = 0$.
Therefore, $\lim_{x\to -3} H(x) = 0$.

33. As inputs x approach 2 from the left, outputs $H(x)$ approach 1. Thus the limit from the left is 1. That is, $\lim_{x\to -2^-} H(x) = 1$.
As inputs x approach 2 from the right, outputs $H(x)$ approach 1. Thus the limit from the right is 1. That is, $\lim_{x\to -2^+} H(x) = 1$.
Since the limit from the left, 1, is the same as the limit from the right, 1, we have $\lim_{x\to -2} H(x) = 1$.

34. As inputs x approach -2 from the right, outputs $H(x)$ approach 1. Thus the limit from the right is 1. That is, $\lim_{x\to -2^+} H(x) = 1$.

35. As inputs x approach 1 from the right, outputs $H(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to -1^+} H(x) = 2$.

36. As inputs x approach 1 from the left, outputs $H(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to 1^-} H(x) = 4$.

37. As inputs x approach 1 from the left, outputs $H(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to 1^-} H(x) = 4$.
As inputs x approach 1 from the right, outputs $H(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to 1^+} H(x) = 2$.
Since the limit from the left, 4, is not the same as the limit from the right, 2, we have $\lim_{x\to 1} H(x)$ *does not exist.*

38. As inputs x approach 3 from the left, outputs $H(x)$ approach 1. Thus the limit from the left is 1. That is, $\lim_{x\to 3^-} H(x) = 1$.

39. As inputs x approach 3 from the left, outputs $H(x)$ approach 1. Thus the limit from the left is 1. That is, $\lim_{x\to 3^-} H(x) = 1$.
As inputs x approach 3 from the right, outputs $H(x)$ approach 1. Thus the limit from the right is 1. That is, $\lim_{x\to 3^+} H(x) = 1$.
Since the limit from the left, 1, is the same as the limit from the right, 1, we have $\lim_{x\to 3} H(x) = 1$.

40. As inputs x approach 3 from the right, outputs $H(x)$ approach 1. Thus the limit from the right is 1. That is,
$\lim_{x\to 3^+} H(x) = 1$.

41. As inputs x approach 2 from the left, outputs $f(x)$ approach -1. Thus the limit from the left is -1. That is, $\lim_{x\to 2^-} f(x) = -1$.
As inputs x approach 2 from the right, outputs $f(x)$ approach -1. Thus the limit from the right is -1. That is, $\lim_{x\to 2^+} f(x) = -1$.
Since the limit from the left, -1, is the same as the limit from the right, -1, we have $\lim_{x\to 2} f(x) = -1$.

42. We have $\lim_{x\to -1^-} f(x) = 1$ and $\lim_{x\to -1^+} f(x) = 1$.
Therefore, $\lim_{x\to -1} f(x) = 1$.

43. As inputs x approach 0 from the left, outputs $f(x)$ approach 2. Thus the limit from the left is 2. That is $\lim_{x\to 0^-} f(x) = 2$.
As inputs x approach 0 from the right, outputs $f(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to 0^+} f(x) = 2$.
Since the limit from the left, 2, is the same as the limit from the right, 2, we have $\lim_{x\to 0} f(x) = 2$.

44. As inputs x approach -3 from the left, outputs $f(x)$ increase without bound. We say that the limit from the left is infinity. That is $\lim_{x\to -3^-} f(x) = \infty$.
As inputs x approach -3 from the right, outputs $f(x)$ decrease without bound. We say that limit from the right is negative infinity. That is, $\lim_{x\to -3^+} f(x) = -\infty$.
Since the function values as $x \to 3$ from the left increase without bound, and the function values as $x \to 3$ from the right decrease without bound, the limit does not exist. We have, $\lim_{x\to -3} f(x)$ *does not exist.*

45. As inputs x approach 1 from the left, outputs $f(x)$ increase without bound. We say that the limit from the left is infinity. That is $\lim_{x\to 1^-} f(x) = \infty$.
As inputs x approach 1 from the right, outputs $f(x)$ decrease without bound. We say that limit from the right is negative infinity. That is, $\lim_{x\to 1^+} f(x) = -\infty$.
Since the function values as $x \to 1$ from the left increase without bound, and the function values as $x \to 1$ from the right decrease without bound, the limit does not exist. We have, $\lim_{x\to 1} f(x)$ *does not exist.*

46. We have $\lim_{x\to 3^-} f(x) = 0$ and $\lim_{x\to 3^+} f(x) = 0$.
Therefore, $\lim_{x\to 3} f(x) = 0$.

47. As inputs x approach 2 from the left, outputs $f(x)$ approach 0. Thus the limit from the left is 0. That is $\lim_{x\to -2^-} f(x) = 0$.
As inputs x approach 2 from the right, outputs $f(x)$ approach 0. Thus the limit from the right is 0. That is, $\lim_{x\to -2^+} f(x) = 0$.
Since the limit from the left, 0, is the same as the limit from the right, 0, we have $\lim_{x\to -2} f(x) = 0$.

48. We have $\lim_{x\to -4^-} f(x) = 3$ and $\lim_{x\to -4^+} f(x) = 3$.
Therefore, $\lim_{x\to -4} f(x) = 3$.

49. As inputs x get more and more negative, output $f(x)$ get closer and closer to 2. $\lim_{x\to -\infty} f(x) = 2$.

50. As inputs x get larger and larger, outputs $f(x)$ get closer and closer to 1. We have $\lim_{x\to \infty} f(x) = 1$.

51. Defining $f(x) = |x|$ as a piecewise defined function we have:

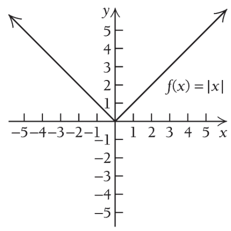
$$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \ge 0. \end{cases}$$

We graph the function by creating an input-output table.

x	-2	-1	0	1	2
$f(x)$	2	1	0	1	2

The solution is continued on the next page.

Next, we plot the points from the table on the previous page and draw the graph.



Find $\lim_{x\to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ approach 0. We have,

$\lim_{x\to 0^-} f(x)=0$.

As inputs x approach 0 from the right, outputs $f(x)$ approach 0. We have,

$\lim_{x\to 0^+} f(x)=0$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to 0} f(x)=0$.

Find $\lim_{x\to -2} f(x)$.

As inputs x approach -2 from the left, outputs $f(x)$ approach 2. We have,

$\lim_{x\to -2^-} f(x)=2$.

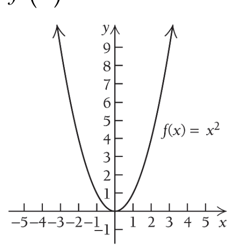
As inputs x approach -2 from the right, outputs $f(x)$ approach 2. We have,

$\lim_{x\to -2^+} f(x)=2$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to -2} f(x)=2$.

52. $f(x)=x^2$



Find $\lim_{x\to -1} f(x)$.

We have $\lim_{x\to -1^-} f(x)=1$ and $\lim_{x\to -1^+} f(x)=1$.

Therefore, $\lim_{x\to -1} f(x)=1$.

Find $\lim_{x\to 0} f(x)$.

We have $\lim_{x\to 0^-} f(x)=0$ and $\lim_{x\to 0^+} f(x)=0$.

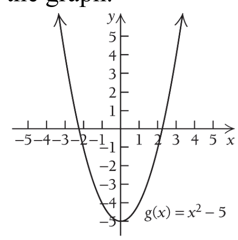
Therefore, $\lim_{x\to 0} f(x)=0$.

53. $g(x)=x^2-5$

We graph the function by creating an input-output table.

x	-2	-1	0	1	2
$g(x)$	-1	-4	-5	-4	-1

Next, we plot the points from the table and draw the graph.



Find $\lim_{x\to 0} g(x)$.

As inputs x approach 0 from the left, outputs $g(x)$ approach -5. We have,

$\lim_{x\to 0^-} g(x)=-5$.

As inputs x approach 0 from the right, outputs $g(x)$ approach -5. We have,

$\lim_{x\to 0^+} g(x)=-5$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to 0} g(x)=-5$.

Find $\lim_{x\to -1} g(x)$.

As inputs x approach -1 from the left, outputs $g(x)$ approach -4. We have,

$\lim_{x\to -1^-} g(x)=-4$.

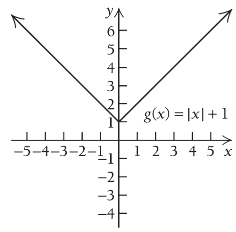
As inputs x approach -1 from the right, outputs $g(x)$ approach -4. We have,

$\lim_{x\to -1^+} g(x)=-4$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to -1} g(x)=-4$.

54. $g(x)=|x|+1$



Find $\lim_{x\to-3} g(x)$.

We have $\lim_{x\to-3^-} g(x)=4$ and $\lim_{x\to-3^+} g(x)=4$.

Therefore, $\lim_{x\to-3} g(x)=4$.

Find $\lim_{x\to 0} g(x)$.

We have $\lim_{x\to 0^-} g(x)=1$ and $\lim_{x\to 0^+} g(x)=1$.

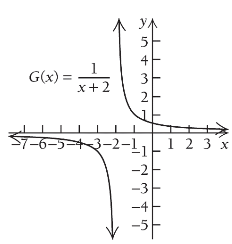
Therefore, $\lim_{x\to 0} g(x)=1$.

55. $G(x)=\dfrac{1}{x+2}$

Since $x=-2$ makes the denominator zero, we exclude the value -2 from the domain.
Creating an input-output table we have

x	-4	-3	-2.1	-1.9	-1	0
$G(x)$	$-\frac{1}{2}$	-1	-10	10	1	$\frac{1}{2}$

Next we plot the points and draw the graph.



Find $\lim_{x\to-1} G(x)$.

As inputs x approach -1 from the left, outputs $G(x)$ approach 1. We have, $\lim_{x\to-1^-} G(x)=1$.

As inputs x approach -1 from the right, outputs $G(x)$ approach 1. We have, $\lim_{x\to-1^+} G(x)=1$. Since the limit from the left is the same as the limit from the right, we have $\lim_{x\to-1} G(x)=1$.

Find $\lim_{x\to-2} G(x)$

As inputs x approach -2 from the left, outputs $G(x)$ decrease without bound . We have,

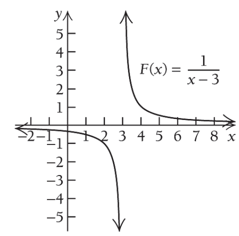
$\lim_{x\to-2^-} G(x)=-\infty$

As inputs x approach -2 from the right, outputs $G(x)$ increase without bound. We have,

$\lim_{x\to-2^+} G(x)=\infty$.

Since the function values as $x\to-2$ from the left decrease without bound, and the function values as $x\to-2$ from the right increase without bound, the limit does not exist. We have, $\lim_{x\to-2} G(x)$ *does not exist*.

56. $F(x)=\dfrac{1}{x-3}$



Find $\lim_{x\to 3} F(x)$.

We have $\lim_{x\to 3^-} F(x)=-\infty$ and $\lim_{x\to 3^+} F(x)=\infty$.

Therefore, $\lim_{x\to 3} F(x)$ *does not exist*.

Find $\lim_{x\to 4} F(x)$.

We have $\lim_{x\to 4^-} F(x)=1$ and $\lim_{x\to 4^+} F(x)=1$

Therefore, $\lim_{x\to 4} F(x)=1$.

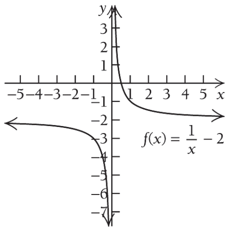
57. $f(x)=\dfrac{1}{x}-2$

Since $x=0$ makes the denominator zero, we exclude the value 0 from the domain. Creating an input-output table we have

x	-1	-0.5	-0.1	0.1	0.5	1
$f(x)$	-3	-4	-12	8	0	-1

The solution is continued on the next page.

Next we plot the points from the table on the previous page and draw the graph.



Find $\lim_{x\to\infty} f(x)$.

As inputs x get larger and larger, outputs $f(x)$ get closer and closer to -2. We have

$\lim_{x\to\infty} f(x) = -2$.

Find $\lim_{x\to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ decrease without bound. We have,

$\lim_{x\to 0^-} f(x) = -\infty$.

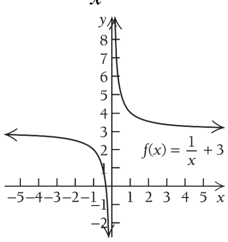
As inputs x approach 0 from the right, outputs $f(x)$ increase without bound. We have,

$\lim_{x\to 0^+} f(x) = \infty$.

Since the function values as $x \to 0$ from the left decrease without bound, and the function values as $x \to 0$ from the right increase without bound, the limit does not exist. We have,

$\lim_{x\to 0} f(x)$ *does not exist.*

58. $f(x) = \dfrac{1}{x} + 3$



Find $\lim_{x\to\infty} f(x)$.

As inputs x get larger and larger, outputs $f(x)$ get closer and closer to 3. We have

$\lim_{x\to\infty} f(x) = 3$.

Find $\lim_{x\to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ decrease without bound. We have,

$\lim_{x\to 0^-} f(x) = -\infty$.

As inputs x approach 0 from the right, outputs $f(x)$ increase without bound. We have,

$\lim_{x\to 0^+} f(x) = \infty$

Since the function values as $x \to 0$ from the left decrease without bound, and the function values as $x \to 0$ from the right increase without bound, the limit does not exist. We have,

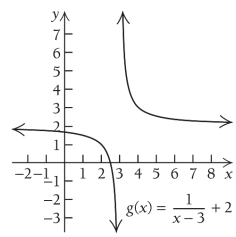
$\lim_{x\to 0} f(x)$ *does not exist.*

59. $g(x) = \dfrac{1}{x-3} + 2$

Since $x = 3$ makes the denominator zero, we exclude the value 3 from the domain. Creating an input-output table we have

x	1	2	2.5	2.9	3.1	3.5	4	5
$g(x)$	$\frac{3}{2}$	1	0	-8	12	4	3	$\frac{5}{2}$

Next we plot the points and draw the graph.



Find $\lim_{x\to\infty} g(x)$.

As inputs x get larger and larger, outputs $g(x)$ get closer and closer to 2. We have

$\lim_{x\to\infty} g(x) = 2$.

Find $\lim_{x\to 3} g(x)$.

As inputs x approach 3 from the left, outputs $g(x)$ decrease without bound. We have,

$\lim_{x\to 3^-} g(x) = -\infty$.

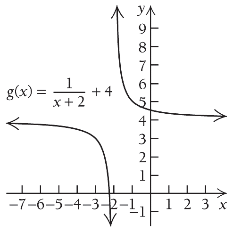
As inputs x approach 3 from the right, outputs $g(x)$ increase without bound. We have,

$\lim_{x\to 3^+} g(x) = \infty$.

Since the function values as $x \to 3$ from the left decrease without bound, and the function values as $x \to 3$ from the right increase without bound, the limit does not exist. We have,

$\lim_{x\to 3} g(x)$ *does not exist.*

60. $g(x) = \dfrac{1}{x+2} + 4$



Find $\lim\limits_{x \to \infty} g(x)$.

As inputs x get larger and larger, outputs $g(x)$ get closer and closer to 4. We have

$\lim\limits_{x \to \infty} g(x) = 4$.

Find $\lim\limits_{x \to -2} g(x)$.

As inputs x approach -2 from the left, outputs $g(x)$ decrease without bound. We have,

$\lim\limits_{x \to -2^-} g(x) = -\infty$.

As inputs x approach -2 from the right, outputs $g(x)$ increase without bound. We have,

$\lim\limits_{x \to -2^+} g(x) = \infty$.

Since the function values as $x \to 2$ from the left decrease without bound, and the function values as $x \to 2$ from the right increase without bound, the limit does not exist. We have,

$\lim\limits_{x \to -2} g(x)$ *does not exist.*

61. $F(x) = \begin{cases} 2x+1, & \text{for } x < 1 \\ x, & \text{for } x \geq 1. \end{cases}$

We create an input-output table for each piece of the function.

For $x < 1$

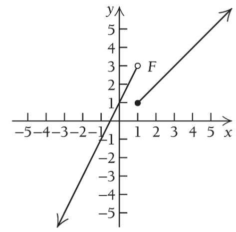
x	-1	0	0.9
$F(x)$	-1	1	2.8

We plot the points and draw the graph. Notice we draw an open circle at the point $(1,3)$ to indicate that the point is not part of the graph.

For $x \geq 1$

x	1	2	3
$F(x)$	1	2	3

We plot the points and draw the graph at the top of the next column. Notice we draw a solid circle at the point $(1,1)$ to indicate that the point is part of the graph.



Find $\lim\limits_{x \to 1^-} F(x)$.

As inputs x approach 1 from the left, outputs $F(x)$ approach 3. That is,

$\lim\limits_{x \to 1^-} F(x) = 3$.

Find $\lim\limits_{x \to 1^+} F(x)$.

As inputs x approach 1 from the right, outputs $F(x)$ approach 1. That is,

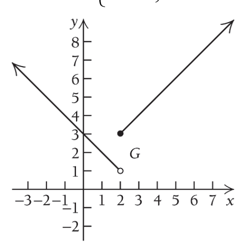
$\lim\limits_{x \to 1^+} F(x) = 1$.

Find $\lim\limits_{x \to 1} F(x)$.

Since the limit from the left, 3, is not the same as the limit from the right, 1, we have

$\lim\limits_{x \to 1} F(x)$ *does not exist.*

62. $G(x) = \begin{cases} -x+3, & \text{for } x < 2 \\ x+1, & \text{for } x \geq 2. \end{cases}$



We have $\lim\limits_{x \to 2^-} G(x) = 1$ and $\lim\limits_{x \to 2^+} G(x) = 3$.

Therefore, $\lim\limits_{x \to 2} G(x)$ *does not exist.*

63. $g(x) = \begin{cases} -x+4, & \text{for } x < 3 \\ x-3, & \text{for } x > 3. \end{cases}$

We create an input-output table for each piece of the function.

For $x < 3$

x	0	1	2	2.9
$g(x)$	4	3	2	1.1

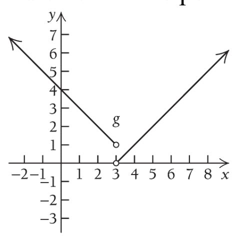
The solution is continued on the next page.

We plot the points from the table on the previous page and draw the graph. Notice we draw an open circle at the point $(3,1)$ to indicate that the point is not part of the graph.

For $x>3$

x	3.1	4	5	6
$g(x)$	0.1	1	2	3

We plot the points and draw the graph. Notice we draw an open circle at the point $(3,0)$ to indicate that the point is not part of the graph.



Find $\lim\limits_{x\to 3^-} g(x)$.

As inputs x approach 3 from the left, outputs $g(x)$ approach 1. That is,

$\lim\limits_{x\to 3^-} g(x)=1$.

Find $\lim\limits_{x\to 3^+} g(x)$.

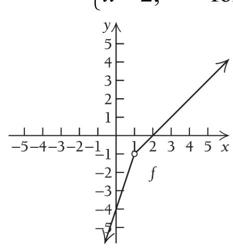
As inputs x approach 3 from the right, outputs $g(x)$ approach 0. That is,

$\lim\limits_{x\to 3^+} g(x)=0$.

Find $\lim\limits_{x\to 3} g(x)$

Since the limit from the left, 1, is not the same as the limit from the right, 0, we have $\lim\limits_{x\to 3} g(x)$ *does not exist.*

64. $f(x)=\begin{cases}3x-4, & \text{for } x<1\\ x-2, & \text{for } x>1.\end{cases}$



We have $\lim\limits_{x\to 1^-} f(x)=-1$ and $\lim\limits_{x\to 1^+} f(x)=-1$.

Therefore, $\lim\limits_{x\to 1} f(x)=-1$.

65. $F(x)=\begin{cases}-2x-3, & \text{for } x<-1\\ x^3, & \text{for } x>-1.\end{cases}$

We create an input-output table for each piece of the function.

For $x<-1$

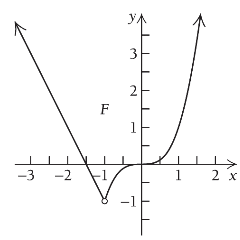
x	–3	–2	–1.1
$F(x)$	3	1	–0.8

We plot the points and draw the graph. Notice we draw an open circle at the point $(-1,-1)$ to indicate that the point is not part of the graph.

For $x>-1$

x	–0.9	0	1
$F(x)$	–0.729	0	1

We plot the points and draw the graph. Notice we draw an open circle at the point $(-1,-1)$ to indicate that the point is not part of the graph.



As inputs x approach -1 from the left, outputs $F(x)$ approach -1. We have,

$\lim\limits_{x\to -1^-} F(x)=-1$.

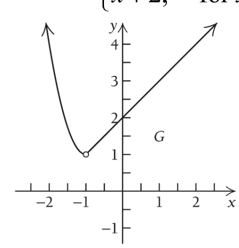
As inputs x approach -1 from the right, outputs $F(x)$ approach -1. We have,

$\lim\limits_{x\to -1^+} F(x)=-1$.

Since the limit from the left is the same as the limit from the right, we have

$\lim\limits_{x\to -1} F(x)=-1$.

66. $G(x)=\begin{cases}x^2, & \text{for } x<-1\\ x+2, & \text{for } x>-1.\end{cases}$



We have, $\lim\limits_{x\to -1^-} G(x)=1$ and $\lim\limits_{x\to -1^+} G(x)=1$.

Therefore, $\lim\limits_{x\to -1} G(x)=1$.

67. $H(x)=\begin{cases} x+1, & \text{for } x<0 \\ 2, & \text{for } 0\le x<1 \\ 3-x, & \text{for } x\ge 1. \end{cases}$

We create an input-output table for each piece of the function.

For $x<0$

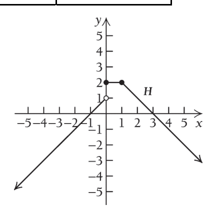
x	-1	-0.5	-0.1
$H(x)$	0	0.5	0.9

We plot the points and draw the graph. Notice we draw an open circle at the point $(0,1)$ to indicate that the point is not part of the graph. For $0\le x<1$, the function has value of 2. We draw a solid circle at the point $(0,2)$ to indicate the point is part of the graph and we draw an open circle at $(1,2)$ to indicate that the point is not part of the graph.

For $x\ge 1$

x	1	2	3
$H(x)$	2	1	0

We plot the points and draw the graph. Notice we draw a solid circle at the point $(1,2)$ to indicate that the point is part of the graph.



Find $\lim\limits_{x\to 0} H(x)$

As inputs x approach 0 from the left, outputs $H(x)$ approach 1. That is,

$\lim\limits_{x\to 0^-} H(x)=1$.

As inputs x approach 0 from the right, outputs $H(x)$ approach 2. That is,

$\lim\limits_{x\to 0^+} H(x)=2$.

Since the limit from the left, 1, is not the same as the limit from the right, 2, we have $\lim\limits_{x\to 0} H(x)$ *does not exist.*

Find $\lim\limits_{x\to 1} H(x)$

As inputs x approach 1 from the left, outputs $H(x)$ approach 2. That is,

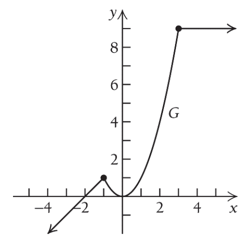
$\lim\limits_{x\to 1^-} H(x)=2$.

As inputs x approach 1 from the right, outputs $H(x)$ approach 2. That is,

$\lim\limits_{x\to 1^+} H(x)=2$.

Since the limit from the left, 2, is the same as the limit from the right, 2, we have $\lim\limits_{x\to 0} H(x)=2$.

68. $G(x)=\begin{cases} 2+x, & \text{for } x\le -1 \\ x^2, & \text{for } -1<x<3 \\ 9, & \text{for } x\ge 3. \end{cases}$



We have $\lim\limits_{x\to -1^-} G(x)=1$ and $\lim\limits_{x\to -1^+} G(x)=1$.

Therefore, $\lim\limits_{x\to -1} G(x)=1$.

We have $\lim\limits_{x\to 3^-} G(x)=9$ and $\lim\limits_{x\to 3^+} G(x)=9$.

Therefore, $\lim\limits_{x\to 3} G(x)=9$.

69. $\lim\limits_{x\to 0.25^-} C(x)=\$3.50$

$\lim\limits_{x\to 0.25^+} C(x)=\$3.50$

$\lim\limits_{x\to 0.25} C(x)=\$3.50$

70. $\lim\limits_{x\to 0.2^-} C(x)=\$3.00$

$\lim\limits_{x\to 0.2^+} C(x)=\$3.50$

$\lim\limits_{x\to 0.2} C(x)$ *does not exist.*

71. $\lim\limits_{x\to 0.6^-} C(x)=\$4.00$.

$\lim\limits_{x\to 0.6^+} C(x)=\$4.50$.

$\lim\limits_{x\to 0.6} C(x)$ *does not exist.*

72. $\lim\limits_{x\to 1^-} p(x)=\$0.98$.

$\lim\limits_{x\to 1^+} p(x)=\$1.19$.

$\lim\limits_{x\to 1} p(x)$ *does not exist.*

73. $\lim\limits_{x\to 2^-} p(x)=\$1.19$.

$\lim\limits_{x\to 2^+} p(x)=\$1.40$.

$\lim\limits_{x\to 2} p(x)$ *does not exist.*

74. $\lim_{x \to 2.6^-} p(x) = \$1.40.$

$\lim_{x \to 2.6^+} p(x) = \$1.40.$

$\lim_{x \to 2.6} p(x) = \$1.40.$

75. $\lim_{x \to 3^-} p(x) = \$1.40.$

$\lim_{x \to 3^+} p(x) = \$1.61.$

$\lim_{x \to 3} p(x)$ *does not exist.*

76. $\lim_{x \to 3.4^-} p(x) = \$1.61.$

$\lim_{x \to 3.4^+} p(x) = \$1.61.$

$\lim_{x \to 3.4} p(x) = \$1.61.$

77. $\lim_{x \to 8925^-} r(x) = 10\%.$

$\lim_{x \to 8925^+} r(x) = 15\%.$

$\lim_{x \to 8925} r(x)$ *does not exist.*

78. $\lim_{x \to 10{,}000^-} r(x) = 15\%.$

$\lim_{x \to 10{,}000^+} r(x) = 15\%.$

$\lim_{x \to 10{,}000} r(x) = 15\%.$

79. $\lim_{x \to 50{,}000^-} r(x) = 25\%.$

$\lim_{x \to 50{,}000^+} r(x) = 25\%.$

$\lim_{x \to 50{,}000} r(x) = 25\%.$

$\lim_{x \to 87{,}850^-} r(x) = 25\%.$

$\lim_{x \to 87{,}850^+} r(x) = 28\%.$

$\lim_{x \to 87{,}850} r(x)$ *does not exist.*

80. $\lim_{x \to 9000^-} r(x) = 10\%.$

$\lim_{x \to 9000^+} r(x) = 10\%.$

$\lim_{x \to 9000} r(x) = 10\%.$

81. $\lim_{x \to 48{,}600^-} r(x) = 15\%.$

$\lim_{x \to 48{,}600^+} r(x) = 25\%.$

$\lim_{x \to 48{,}600} r(x)$ *does not exist.*

82. $\lim_{x \to 60{,}000^-} r(x) = 25\%.$

$\lim_{x \to 60{,}000^+} r(x) = 25\%.$

$\lim_{x \to 60{,}000} r(x) = 25\%.$

$\lim_{x \to 12{,}570^-} r(x) = 10\%.$

$\lim_{x \to 12{,}570^+} r(x) = 15\%.$

$\lim_{x \to 12{,}570} r(x)$ *does not exist.*

83. As inputs x approach 2 from the right, outputs $f(x)$ approach 4. We have, $\lim_{x \to 2^+} f(x) = 4$. In order for $\lim_{x \to 2} f(x)$ to exist we need $\lim_{x \to 2^-} f(x) = 4$. We will use the letter c for the unknown in the equation; therefore,

$\lim_{x \to 2^-} \frac{1}{2}(x) + c = 4$.

Substitute 2 in for x to get the equation:

$\frac{1}{2}(2) + c = 4$ and solving for c we get

$1 + c = 4$

$c = 3.$

Therefore, in order for the limit to exist as x approaches 2, the function must be:

$$f(x) = \begin{cases} \frac{1}{2}x + \underline{3} & \text{for } x < 2 \\ -x + 6 & \text{for } x > 2. \end{cases}$$

84. As inputs x approach 2 from the left, outputs $f(x)$ approach 0. We have, $\lim_{x \to 2^-} f(x) = 0$. In order for $\lim_{x \to 2} f(x)$ to exist we need $\lim_{x \to 2^+} f(x) = 0$. We will use the letter c for the unknown in the equation and this gives us

$\lim_{x \to 2^+} \frac{3}{2}(x) + c = 0$

Substitute 2 in for x to get the equation:

$\frac{3}{2}(2) + c = 0$. We solve for c on the next page.

Solving the equation on the previous page for c we have:

$3+c=0$

$c=-3.$

Therefore, in order for the limit to exist as x approaches 2, the function must be:

$$f(x)=\begin{cases}-\frac{1}{2}x+1 & \text{for } x<2\\ \frac{3}{2}x+\underline{-3} & \text{for } x>2.\end{cases}$$

85. As inputs x approach 2 from the left, outputs $f(x)$ approach -5. We have, $\lim_{x\to 2^-} f(x)=-5$. In order for $\lim_{x\to 2} f(x)$ to exist we need $\lim_{x\to 2^+} f(x)=-5$. We will use the letter c for the unknown in the equation and this gives us

$\lim_{x\to 2^+}\left(-x^2+c\right)=-5$

Substitute 2 in for x to get the equation:

$-(2)^2+c=-5$ and solving for c we get

$-(4)+c=-5$

$c=-1.$

Therefore, in order for the limit to exist as x approaches 2, the function must be:

$$f(x)=\begin{cases}x^2-9 & \text{for } x<2\\ -x^2+\underline{-1} & \text{for } x>2.\end{cases}$$

86. Graph $f(x)=\begin{cases}-3, & \text{for } x=-2\\ x^2, & \text{for } x\neq -2.\end{cases}$

Using the calculator we enter the function into the graphing editor as follows:

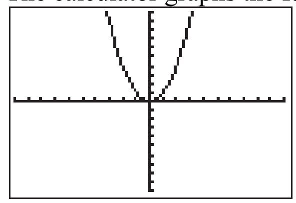
```
Plot1 Plot2 Plot3
\Y1■-3/(X=-2)
\Y2■X^2/(X≠-2)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Notice, when you select the table feature you get:

X	Y1	Y2
-6	ERROR	36
-5	ERROR	25
-4	ERROR	16
-3	ERROR	9
-2	-3	ERROR
-1	ERROR	1
0	ERROR	0

X=0

The calculator graphs the function:



Using the trace feature, we find the limits.

a) $\lim_{x\to -2^+} f(x)=4$

b) $\lim_{x\to -2^-} f(x)=4$

c) $\lim_{x\to -2} f(x)=4$

d) $\lim_{x\to 2^+} f(x)=4$

e) $\lim_{x\to 2^-} f(x)=4$

f) $\lim_{x\to -2} f(x)=4\neq -3=f(-2)$

g) $\lim_{x\to 2} f(x)=4=f(2)$

87. Graph $f(x)=\begin{cases}x^2-2, & \text{for } x<0\\ 2-x^2, & \text{for } x\geq 0.\end{cases}$

Using the calculator we enter the function into the graphing editor as follows:

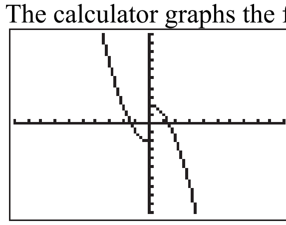
```
Plot1 Plot2 Plot3
\Y1■X^2-2/(X<0)
\Y2■2-X^2/(X≥0)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

When you select the table feature you get:

X	Y1	Y2
-3	7	ERROR
-2	2	ERROR
-1	-1	ERROR
0	ERROR	2
1	ERROR	1
2	ERROR	-2
3	ERROR	-7

X=3

The calculator graphs the function



Using the trace feature, we find the limits.

Find $\lim_{x\to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ approach -2. We have,

$\lim_{x\to 0^-} f(x)=-2$.

The solution is continued on the next page.

As inputs x approach 0 from the right, outputs $f(x)$ approach 2. We have,

$$\lim_{x\to 0^+} f(x) = 2$$

Since the limit from the left, -2, is not the same as the limit from the right, 2, we have

$\lim_{x\to 0} f(x)$ *does not exist.*

Find $\lim_{x\to -2} f(x)$.

As inputs x approach -2 from the left, outputs $f(x)$ approach 2. We have,

$$\lim_{x\to -2^-} f(x) = 2.$$

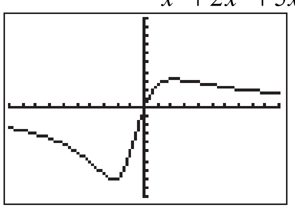
As inputs x approach -2 from the right, outputs $f(x)$ approach 2. We have,

$$\lim_{x\to -2^+} f(x) = 2$$

Since the limit from the left is the same as the limit from the right, we have

$$\lim_{x\to -2} f(x) = 2.$$

88. Graph $g(x) = \dfrac{20x^2}{x^3 + 2x^2 + 5x}$



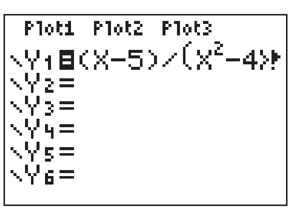
Using the trace feature on the graph, we have:

$$\lim_{x\to \infty} g(x) = 0$$

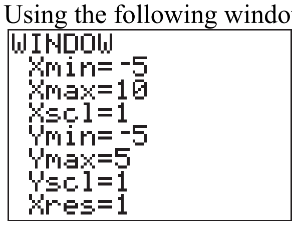
$$\lim_{x\to -\infty} g(x) = 0.$$

89. Graph $f(x) = \dfrac{x-5}{x^2 - 4x - 5}$

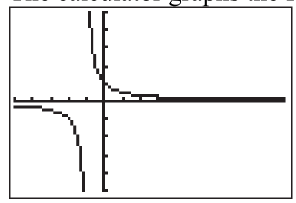
Using the calculator we enter the function into the graphing editor.



Using the following window:



The calculator graphs the function:



Using the trace feature on the calculator we find the limits.

Find $\lim_{x\to -1} f(x)$.

As inputs x approach -1 from the left, outputs $f(x)$ increase without bound. We have,

$$\lim_{x\to -1^-} f(x) = -\infty.$$

As inputs x approach -1 from the right, outputs $f(x)$ decrease without bound. We have,

$$\lim_{x\to -1^+} f(x) = \infty$$

Since the function values as $x \to -1$ from the left increase without bound, and the function values as $x \to -1$ from the right decrease without bound, the limit does not exist.

$\lim_{x\to -1} f(x)$ *does not exist.*

Find $\lim_{x\to 5} f(x)$.

As inputs x approach 5 from the left, outputs $f(x)$ approach $\frac{1}{6}$. We have,

$$\lim_{x\to 5^-} f(x) = \frac{1}{6}.$$

As inputs x approach 5 from the right, outputs $f(x)$ approach $\frac{1}{6}$. We have,

$$\lim_{x\to 5^+} f(x) = \frac{1}{6}.$$

Since the limit from the left is the same as the limit from the right, we have

$$\lim_{x\to 5} f(x) = \frac{1}{6}.$$

Exercise Set 1.2

1. By limit property L1, $\lim_{x\to 3} 7 = 7$.
Therefore, the statement $\lim_{x\to 3} 7 = 3$ is a false.

2. By limit property L2, the statement is true.

3. By limit property L2,
$\lim_{x\to 1}\left[g(x)\right]^2 = \left[\lim_{x\to 1} g(x)\right]^2 = [5]^2 = 25.$
Therefore, the statement is true.

4. By limit property L6, the statement is true.

5. The statement is false. $g(3)$ could exist and if $\lim_{x\to 3} g(x)$ *does not exist*, then the function would still be discontinuous at $x = 3$. .

6. By the definition of continuity, in order for *f* to be continuous at $x = 2$, $f(2)$ must exist. Therefore, the statement is true.

7. This statement is false. If $\lim_{x\to 4} F(x)$ exists but is not equal to $F(4)$, then F is not continuous.

8. By the definition of continuity, the statement is true.

9. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$\lim_{x\to 2}(4x-5) = 4(2)-5 = 3.$

10. $\lim_{x\to 1}(3x+2) = 3(1)+2 = 5.$

11. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\begin{aligned}\lim_{x\to -1}\left(x^2-4\right) &= (-1)^2-4\\ &= 1-4\\ &= -3.\end{aligned}$$

12. $\lim_{x\to -2}\left(x^2+3\right) = (-2)^2+3 = 7.$

13. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\begin{aligned}\lim_{x\to 5}\left(x^2-6x+9\right) &= (5)^2-6(5)+9\\ &= 25-30+9\\ &= 4.\end{aligned}$$

14. $\lim_{x\to 3}\left(x^2-4x+7\right) = (3)^2-4(3)+7 = 4.$

15. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\begin{aligned}&\lim_{x\to 2}\left(2x^4-3x^3+4x-1\right)\\ &= 2(2)^4-3(2)^3+4(2)-1\\ &= 2(16)-3(8)+8-1\\ &= 32-24+8-1\\ &= 15.\end{aligned}$$

16.
$$\begin{aligned}&\lim_{x\to -1}\left(3x^5+4x^4-3x+6\right)\\ &= 3(-1)^5+4(-1)^4-3(-1)+6\\ &= 10.\end{aligned}$$

17. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\begin{aligned}\lim_{x\to 3}\frac{x^2-25}{x^2-5} &= \frac{(3)^2-25}{(3)^2-5}\\ &= \frac{9-25}{9-5}\\ &= \frac{-16}{4}\\ &= -4\end{aligned}$$

18. $\lim_{x\to 3}\left(\frac{x^2-8}{x-2}\right) = \frac{(3)^2-8}{3-2} = \frac{9-8}{3-2} = 1.$

19. We verify the expression yields an indeterminate form by substitution:
$$\begin{aligned}\lim_{x\to 3}\frac{x^2-9}{x-3} &= \frac{(3)^2-9}{(3)-3}\\ &= \frac{0}{0}.\end{aligned}$$
This is an indeterminate form.
The solution is continued on the next page.

In order to find the limit we will simplify the function by factoring the numerator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\begin{aligned}\lim_{x\to 3}\frac{x^2-9}{x-3} &= \lim_{x\to 3}\frac{(x-3)(x+3)}{x-3}\\ &= \lim_{x\to 3}(x+3) && \text{simplifying, assuming } x\neq 3\\ &= 3+3 && \text{substitution}\\ &= 6.\end{aligned}$$

20. $\lim_{x\to 5}\frac{x^2-25}{x-5}$

First we will simplify the function by factoring the numerator and canceling common factors.

$$\begin{aligned}\lim_{x\to 5}\frac{x^2-25}{x-5} &= \lim_{x\to 5}\frac{(x-5)(x+5)}{x-5}\\ &= \lim_{x\to 5}(x+5)\\ &= 5+5\\ &= 10.\end{aligned}$$

21. We verify the expression yields an indeterminate form by substitution:

$$\begin{aligned}\lim_{x\to -2}\frac{x^2-2x-8}{x^2-4} &= \frac{(-2)^2-2(-2)-8}{(-2)^2-4}\\ &= \frac{0}{0}.\end{aligned}$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the numerator and denominator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\begin{aligned}\lim_{x\to -2}\frac{x^2-2x-8}{x^2-4} &= \lim_{x\to -2}\frac{(x-4)(x+2)}{(x-2)(x+2)}\\ &= \lim_{x\to -2}\left(\frac{x-4}{x-2}\right) && \text{simplifying, assuming } x\neq -2\\ &= \frac{-2-4}{-2-2} && \text{substitution}\\ &= \frac{-6}{-4}\\ &= \frac{3}{2}.\end{aligned}$$

22. $\lim_{x\to 1}\frac{x^2+5x-6}{x^2-1}$

First we will simplify the function by factoring the numerator and denominator then canceling common factors.

$$\begin{aligned}\lim_{x\to 1}\frac{x^2+5x-6}{x^2-1} &= \lim_{x\to 1}\frac{(x-1)(x+6)}{(x-1)(x+1)}\\ &= \lim_{x\to 1}\left(\frac{x+6}{x+1}\right) && \text{simplifying, assuming } x\neq 1\\ &= \frac{1+6}{1+1} && \text{substitution}\\ &= \frac{7}{2}.\end{aligned}$$

23. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 2}\frac{3x^2+x-14}{x^2-4} = \frac{3(2)^2+(2)-14}{(2)^2-4} = \frac{0}{0}.$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the numerator and denominator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\begin{aligned}\lim_{x\to 2}\frac{3x^2+x-14}{x^2-4} &= \lim_{x\to 2}\frac{(3x+7)(x-2)}{(x+2)(x-2)}\\ &= \lim_{x\to 2}\left(\frac{3x+7}{x+2}\right) && \text{simplifying, assuming } x\neq 2\\ &= \frac{3(2)+7}{2+2} && \text{substitution}\\ &= \frac{13}{4}.\end{aligned}$$

24. $\lim_{x\to -3}\frac{2x^2-x-21}{9-x^2}$

First we will simplify the function by factoring the numerator and denominator and canceling common factors.

$$\begin{aligned}\lim_{x\to -3}\frac{2x^2-x-21}{9-x^2} &= \lim_{x\to -3}\frac{(2x-7)(x+3)}{(3-x)(3+x)}\\ &= \lim_{x\to -3}\left(\frac{2x-7}{3-x}\right)\\ &= \frac{2(-3)-7}{3-(-3)}\\ &= -\frac{13}{6}.\end{aligned}$$

25. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 2}\frac{x^3-8}{2-x}=\frac{(2)^3-8}{2-2}=\frac{0}{0}.$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the numerator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\lim_{x\to 2}\frac{x^3-8}{2-x}=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{-(x-2)}$$

$$=\lim_{x\to 2}\left[-(x^2+2x+4)\right] \quad \text{simplifying, assuming } x\neq 2$$

$$=-\left((2)^2+2(2)+4\right) \quad \text{substitution}$$

$$=-12.$$

26. $\lim_{x\to 1}\frac{x^3-1}{x-1}$

First we will simplify the function by factoring the numerator and canceling common factors.

$$\lim_{x\to 1}\frac{x^3-1}{x-1}=\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)}$$

$$=\lim_{x\to 1}(x^2+x+1) \quad \text{simplifying, assuming } x\neq 1$$

$$=(1)^2+(1)+1=3. \quad \text{substitution}$$

27. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}=\frac{\sqrt{25}-5}{(25)-25}=\frac{0}{0}.$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the denominator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}=\lim_{x\to 25}\frac{\sqrt{x}-5}{(\sqrt{x}-5)(\sqrt{x}+5)}$$

$$=\lim_{x\to 25}\frac{1}{(\sqrt{x}+5)} \quad \text{simplifying, assuming } x\neq 25$$

$$=\frac{1}{\sqrt{25}+5} \quad \text{substitution}$$

$$=\frac{1}{10}.$$

28. $\lim_{x\to 9}\frac{9-x}{\sqrt{x}-3}$

First we will simplify the function by factoring the numerator and canceling common factors.

$$\lim_{x\to 9}\frac{9-x}{\sqrt{x}-3}=\lim_{x\to 9}\frac{-(x-9)}{\sqrt{x}-3}$$

$$=\lim_{x\to 9}\left[\frac{-(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3}\right]$$

$$=\lim_{x\to 9}\left[-(\sqrt{x}+3)\right]$$

$$=-(\sqrt{9}+3)$$

$$=-6.$$

29. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 2}\frac{x^2+3x-10}{x^2-4x+4}=\frac{(2)^2+3(2)-10}{(2)^2-4(2)+4}$$

$$=\frac{0}{0}.$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the numerator and the denominator then canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\lim_{x\to 2}\frac{x^2+3x-10}{x^2-4x+4}=\lim_{x\to 2}\frac{(x-2)(x+5)}{(x-2)(x-2)}$$

$$=\lim_{x\to 2}\left(\frac{x+5}{x-2}\right) \quad \text{simplifying, assuming } x\neq 2$$

$$=\frac{2+5}{2-2} \quad \text{substitution}$$

$$=\frac{7}{0}.$$

Substitution yields division by zero. Therefore,

$\lim_{x\to 2}\frac{x^2+3x-10}{x^2-4x+4}$ *does not exist.*

30. $\lim_{x\to -1} \frac{x^2+5x+4}{x^2+2x+1}$

First we will simplify the function by factoring the numerator and denominator then canceling common factors.

$$\lim_{x\to -1} \frac{x^2+5x+4}{x^2+2x+1} = \lim_{x\to -1} \frac{(x+1)(x+4)}{(x+1)(x+1)}$$
$$= \lim_{x\to -1}\left(\frac{x+4}{x+1}\right) \quad \text{simplifying, assuming } x\neq -1$$
$$= \frac{-1+4}{-1+1} \quad \text{substitution}$$
$$= \frac{3}{0}.$$

Substitution yields division by zero. Therefore, $\lim_{x\to -1} \frac{x^2+5x+4}{x^2+2x+1}$ *does not exist.*

31. $\lim_{x\to 5} \sqrt{x^2-16}$

By limit property L2,

$$\lim_{x\to 5} \sqrt{x^2-16} = \sqrt{\lim_{x\to 5}\left(x^2-16\right)}$$
$$= \sqrt{\lim_{x\to 5} x^2 - \lim_{x\to 5} 16} \quad \text{By L3}$$
$$= \sqrt{\left(\lim_{x\to 5} x\right)^2 - 16} \quad \text{By L2 and L1}$$
$$= \sqrt{(5)^2-16}$$
$$= \sqrt{25-16}$$
$$= \sqrt{9}$$
$$= 3.$$

32. $\lim_{x\to 4} \sqrt{x^2-9}$

By limit property L2,

$$\lim_{x\to 4} \sqrt{x^2-9} = \sqrt{\lim_{x\to 4}\left(x^2-9\right)}$$
$$= \sqrt{\lim_{x\to 4} x^2 - \lim_{x\to 4} 9} \quad \text{By L3}$$
$$= \sqrt{\left(\lim_{x\to 4} x\right)^2 - 9} \quad \text{By L2 and L1}$$
$$= \sqrt{(4)^2-9}$$
$$= \sqrt{16-9}$$
$$= \sqrt{7}.$$

33. $\lim_{x\to 2} \sqrt{x^2-9}$

By limit property L2,

$$\lim_{x\to 2} \sqrt{x^2-9} = \sqrt{\lim_{x\to 2}\left(x^2-9\right)}$$
$$= \sqrt{\lim_{x\to 2} x^2 - \lim_{x\to 2} 9} \quad \text{By L3}$$
$$= \sqrt{\left(\lim_{x\to 2} x\right)^2 - 9} \quad \text{By L2 and L1}$$
$$= \sqrt{(2)^2-9}$$
$$= \sqrt{4-9}$$
$$= \sqrt{-5}.$$

Therefore, $\lim_{x\to 2} \sqrt{x^2-9}$ *does not exist.*

34. $\lim_{x\to 3} \sqrt{x^2-16} = \sqrt{\lim_{x\to 3}\left(x^2-16\right)}$ By L2

$$= \sqrt{\lim_{x\to 3} x^2 - \lim_{x\to 3} 16} \quad \text{By L3}$$
$$= \sqrt{\left(\lim_{x\to 3} x\right)^2 - 16} \quad \text{By L2 and L1}$$
$$= \sqrt{(3)^2-16}$$
$$= \sqrt{9-16}$$
$$= \sqrt{-7}.$$

Therefore, $\lim_{x\to 3} \sqrt{x^2-16}$ *does not exist.*

35. $\lim_{x\to -4^-} \sqrt{x^2-16}$

By limit property L2,

$$\lim_{x\to -4^-} \sqrt{x^2-16} = \sqrt{\lim_{x\to -4^-}\left(x^2-16\right)}$$
$$= \sqrt{\lim_{x\to -4^-} x^2 - \lim_{x\to -4^-} 16} \quad \text{By L3}$$
$$= \sqrt{\left(\lim_{x\to -4^-} x\right)^2 - 16} \quad \text{By L2 and L1}$$
$$= \sqrt{(-4)^2-16}$$
$$= \sqrt{16-16}$$
$$= \sqrt{0}$$
$$= 0.$$

36. $\lim_{x\to 3^+}\sqrt{x^2-9}=\sqrt{\lim_{x\to 3^+}(x^2-9)}$ By L2

$=\sqrt{\lim_{x\to 3^+}x^2-\lim_{x\to 3^+}9}$ By L3

$=\sqrt{\left(\lim_{x\to 3^+}x\right)^2-9}$ By L2 and L1

$=\sqrt{(3)^2-9}$

$=\sqrt{9-9}$

$=\sqrt{0}$

$=0.$

37. The function is not continuous over the interval, because $g(x)$ is not continuous at $x=-2$. As x approaches -2 from the left, $g(x)$ approaches 4. However, as x approaches -2 from the right $f(x)$ approaches -3. Therefore, $g(x)$ is not continuous at -2.

38. The function is not continuous over the interval, because it is not continuous at $x=1$.

39. The function is continuous over the interval.

40. The function is not continuous over the interval, because $k(x)$ is not continuous at $x=-1$. The function is not defined at $x=-1$, in other words $k(-1)$ does not exist. Therefore, $k(x)$ is not continuous at -1.

41. The function is not continuous over the interval, because it is not continuous at $x=-2$. We see that $\lim_{x\to -2}t(x)=\infty$, therefore, the limit does not exist as x approaches -2; furthermore, the function is not defined at $x=-2$, in other words $t(-2)$ does not exist. Therefore the function is not continuous at $x=-2$.

42. a) We have $\lim_{x\to 1^+}g(x)=-2$ and $\lim_{x\to 1^-}g(x)=-2.$ Therefore, $\lim_{x\to 1}g(x)=-2$.

b) $g(1)=-2$.

c) The function $g(x)$ is continuous at $x=1$, because
1) $g(1)$ exists, $g(1)=-2$,

2) $\lim_{x\to 1}g(x)$ exists, $\lim_{x\to 1}g(x)=-2$, and
3) $\lim_{x\to 1}g(x)=-2=g(1)$.

d) $\lim_{x\to -2^+}g(x)=-3$ and $\lim_{x\to -2^-}g(x)=4$. Therefore, $\lim_{x\to -2}g(x)$ does not exist.

e) $g(-2)=-3$.

f) Since the limit of $g(x)$ as x approaches -2 does not exist, the function is not continuous at $x=-2$.

43. a) As inputs x approach 1 from the right, outputs $f(x)$ approach -1. Thus, the limit from the right is -1. $\lim_{x\to 1^+}f(x)=-1.$
As inputs x approach 1 from the left, outputs $f(x)$ approach 2. Thus, the limit from the left is 2. $\lim_{x\to 1^-}f(x)=2$.
Since the limit from the left, -1, is not the same as the limit from the right, 2, the $\lim_{x\to 1}f(x)$ *does not exist*

b) When the input is 1, the output $f(1)$ is -1. That is $f(1)=-1$

c) Since the limit of $f(x)$ as x approaches 1 does not exist, the function is not continuous at $x=1$.

d) As inputs x approach -2 from the right, outputs $f(x)$ approach 3. Thus, the limit from the right is 3. $\lim_{x\to -2^+}f(x)=3$.
As inputs x approach -2 from the left, outputs $f(x)$ approach 3. Thus, the limit from the left is 3. $\lim_{x\to -2^-}f(x)=3$.
Since the limit from the left, 3, is the same as the limit from the right, 3, we say $\lim_{x\to -2}f(x)=3.$.

e) When the input is -2, the output $f(-2)$ is 3. That is $f(-2)=3$

f) The function $f(x)$ is continuous at $x=-2$, because
1) $f(-2)$ exists, $f(-2)=3$,
2) $\lim_{x\to -2} f(x)$ exists, $\lim_{x\to -2} f(x)=3$, and
3) $\lim_{x\to -2} f(x)=3=f(-2)$.

44. a) $\lim_{x\to 1^+} h(x)=2$ and $\lim_{x\to 1^-} h(x)=2$.
Therefore, $\lim_{x\to 1} h(x)=2$.

b) $h(1)=2$.

c) The function $h(x)$ is continuous at $x=1$, because
1) $h(1)$ exists, $h(1)=2$,
2) $\lim_{x\to 1} h(x)$ exists, $\lim_{x\to 1} h(x)=2$, and
3) $\lim_{x\to 1} h(x)=2=h(1)$.

d) $\lim_{x\to -2^+} h(x)=0$ and $\lim_{x\to -2^-} h(x)=0$.
Therefore, $\lim_{x\to -2} h(x)=0$.

e) $h(-2)=0$.

f) The function $h(x)$ is continuous at $x=-2$, because
1) $h(-2)$ exists, $h(-2)=0$,
2) $\lim_{x\to -2} h(x)$ exists, $\lim_{x\to -2} h(x)=0$, and
3) $\lim_{x\to -2} h(x)=0=h(-2)$.

45. a) As inputs x approach -1 from the right, outputs $k(x)$ approach 2. Thus, the limit from the right is 2. $\lim_{x\to -1^+} k(x)=2$.
As inputs x approach -1 from the left, outputs $k(x)$ approach 2. Thus, the limit from the left is 2. $\lim_{x\to -1^-} k(x)=2$.
Since the limit from the left, 2, is the same as the limit from the right, 2, we have $\lim_{x\to -1} k(x)=2$.

b) $k(-1)$ is not defined, or does not exist.

c) Since the value of $k(-1)$ does not exist, the function is not continuous at $x=-1$.

d) As inputs x approach 3 from the right, outputs $k(x)$ approach -2. Thus, the limit from the right is -2. $\lim_{x\to 3^+} k(x)=-2$.
As inputs x approach 3 from the left, outputs $k(x)$ approach -2. Thus, the limit from the left is -2. $\lim_{x\to 3^-} k(x)=-2$.
Since the limit from the left, -2, is the same as the limit from the right, -2, we have $\lim_{x\to 3} k(x)=-2$.

e) $k(3)=-2$

f) The function $k(x)$ is continuous at $x=3$, because
1) $k(3)$ exists,
2) $\lim_{x\to 3} k(x)$ exists, and
3) $\lim_{x\to 3} k(x)=-2=k(3)$.

46. a) $\lim_{x\to 1^+} t(x)=0.25$ and $\lim_{x\to 1^-} t(x)=0.25$.
Therefore, $\lim_{x\to 1} t(x)=0.25$.

b) $t(1)=0.25$

c) The function $t(x)$ is continuous at $x=1$, because
1) $t(1)$ exists,
2) $\lim_{x\to 1} t(x)$ exists, and
3) $\lim_{x\to 1} t(x)=0.25=t(1)$.

d) $\lim_{x\to -2^+} t(x)=\infty$ and $\lim_{x\to -2^-} t(x)=\infty$
Therefore, $\lim_{x\to -2} t(x)=\infty$, which means that the limit does not exist because as x gets closer to -2 the function values increase without bound.

e) $t(-2)$ is undefined or does not exist.

f) Since $t(-2)$ is undefined and the limit of $t(x)$ as x approaches -2 does not exist, the function is not continuous at $x=-2$.

47. a) As inputs x approach 3 from the right, outputs $G(x)$ approach 3. Thus, $\lim_{x\to 3^+} G(x)=3$.

b) As inputs x approach 3 from the left, outputs $G(x)$ approach 1. Thus, $\lim_{x\to 3^-} G(x)=1$.

c) Since the limit from the left, 1, is not the same as the limit from the right, 3, the limit does not exist. $\lim_{x\to 3} G(x)$ does not exist.

d) $G(3)=1$

e) The function $G(x)$ is not continuous at $x=3$ because the limit does not exist as x approaches 3.

f) The function $G(x)$ is continuous at $x=0$, because
1) $G(0)$ exists,
2) $\lim_{x\to 0} G(x)$ exists, and
3) $\lim_{x\to 0} G(x) = G(0)$.

g) The function $G(x)$ is continuous at $x=2.9$, because
1) $G(2.9)$ exists,
2) $\lim_{x\to 2.9} G(x)$ exists, and
3) $\lim_{x\to 2.9} G(x) = G(2.9)$.

48. a) $\lim_{x\to 2^+} C(x) = 1$

b) $\lim_{x\to 2^-} C(x) = -1$

c) $\lim_{x\to 2} C(x)$ does not exist.

d) $C(2)=1$

e) The function $C(x)$ is not continuous at $x=2$ because the limit does not exist as x approaches 2.

f) The function $C(x)$ is continuous at $x=1.95$, because $\lim_{x\to 1.95} C(x) = -1 = C(1.95)$.

49. First we find the function value when $x=4$. $g(4)=(4)^2-3(4)=4$. Hence, $g(4)$ exists. Next, we find the limit as x approaches 4. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution: $\lim_{x\to 4} g(x) = (4)^2 - 3(4) = 4$.
Therefore, $\lim_{x\to 4} g(x) = 4 = g(4)$ and the function is continuous at $x = 4$.

50. The function $f(x)$ is continuous at $x=5$, because:
1) $f(5)$ exists, $f(5)=13$
2) $\lim_{x\to 5} f(x)$ exists, $\lim_{x\to 5} f(x) = 13$, and
3) $\lim_{x\to 5} f(x) = 13 = f(5)$.

51. The function $G(x)=\frac{1}{x}$ is not continuous at $x=0$ because $G(0)=\frac{1}{0}$ is undefined.

52. The function $F(x)=\sqrt{x}$ is not continuous at $x=-1$ because $F(-1)=\sqrt{-1}$ is undefined on the real numbers.

53. First we find the function value when $x=4$. $f(4)=-(4)+7=3$. Hence, $f(4)$ exists. Next we find the limit as x approaches 4. As the inputs x approach 4 from the left, the outputs $f(x)$ approach 3, that is,

$\lim_{x\to 4^-} f(x) = \frac{1}{2}(4)+1 = 3$.

As the inputs x approach 4 from the right, the outputs $f(x)$ approach 3, that is,

$\lim_{x\to 4^+} f(x) = -(4)+7 = 3$.

Since the limit from the left, 3, is the same as the limit from the right, 3. The limit exists. We have:

$\lim_{x\to 4} f(x) = 3$.

Therefore, we have $\lim_{x\to 4} f(x) = 3 = f(4)$.
Thus the function is continuous at $x = 4$.

54. The function $f(x)$ is continuous at $x=3$, because:
1) $g(3)$ exists, $g(3)=5$
2) $\lim_{x\to 3} g(x)$ exists, $\lim_{x\to 3} g(x) = 5$, and
3) $\lim_{x\to 3} g(x) = 5 = g(3)$.

55. The function is not continuous at $x=3$ because the limit does not exist as x approaches 3. To verify this we take the limit as x approaches 3 from the left and the limit as x approaches 3 from the right.
The solution is continued on the next page.

As x approaches 3 from the left we have
$\lim_{x\to 3^-} F(x) = \frac{1}{3}(3)+4=5$.
As x approaches 3 from the right we have
$\lim_{x\to 3^+} F(x) = 2(3)-5=1$.
Since the limit from the left, 5, is not the same as the limit from the right, 1, the limit does not exist. $\lim_{x\to 3} F(x)$ does not exist.

56. The function is not continuous at $x=4$ because the limit does not exist as x approaches 4.
$\lim_{x\to 4^-} G(x) = \frac{1}{2}(4)+1=3$
$\lim_{x\to 4^+} G(x) = -(4)+5=1$
$\lim_{x\to 4^-} G(x) \neq \lim_{x\to 4^+} G(x)$
Therefore,
$\lim_{x\to 4} G(x)$ does not exist.
Furthermore, the function is not defined at $x=4$, so $G(4)$ does not exist.

57. The function is not continuous at $x=4$ because the function is not defined at $x=4$. Therefore, $g(4)$ does not exist.

58. The function $f(x)$ is continuous at $x=3$, because:
1) $f(3)$ exists, $f(3)=5$
2) $\lim_{x\to 3} f(x)$ exists, $\lim_{x\to 3} f(x)=5$, and
3) $\lim_{x\to 3} f(x)=5=f(3)$.

59. The function is not continuous at $x=2$. To verify this, we take the limit as x approaches 2. Using the Theorem on Limits of Rational Functions, we simplify the function near 2 by factoring the numerator and canceling common factors.

$$\begin{aligned}\lim_{x\to 2} G(x) &= \lim_{x\to 2}\frac{x^2-4}{x-2}\\ &= \lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}\\ &= \lim_{x\to 2}(x+2)\\ &= 2+2=4\end{aligned}$$

Therefore,
$\lim_{x\to 2} G(x)=4$.

However, when $x=2$, the output $G(2)$ is defined to be 5. That is, $G(2)=5$. Therefore, $\lim_{x\to 2} G(x)=4\neq 5=G(2)$. Thus the function is not continuous at $x=2$.

60. The function is not continuous at $x=1$ because $\lim_{x\to 1} F(x)=2\neq 4=F(1)$.

61. First we find the function value when $x=4$.
$G(4)=2(4)-3=5$, $G(4)$ exists.
Next we find the limit as x approaches 4. To find the limit as x approaches 4 from the left, we first simplify the rational function by factoring the numerator and canceling common factors.

$$\begin{aligned}\lim_{x\to 4^-} G(x) &= \lim_{x\to 4^-}\frac{x^2-3x-4}{x-4}\\ &= \lim_{x\to 4^-}\frac{(x-4)(x+1)}{x-4}\\ &= \lim_{x\to 4^-}(x+1)\\ &= 4+1\\ &= 5\end{aligned}$$

To find the limit as x approaches 4 from the right, we can use substitution.

$$\begin{aligned}\lim_{x\to 4^+} G(x) &= \lim_{x\to 4^+}(2x-3)\\ &= 2(4)-3\\ &= 5\end{aligned}$$

Therefore, the limit exists.
$\lim_{x\to 4} G(x)=5$.
Thus we have,
$\lim_{x\to 4} G(x)=5=G(4)$.
Therefore, the function in continuous at $x=4$.

62. The function $f(x)$ is continuous at $x=5$, because:
1) $f(5)$ exists, $f(5)=6$
2) $\lim_{x\to 5} f(x)$ exists, $\lim_{x\to 5} f(x)=6$, and
3) $\lim_{x\to 5} f(x)=6=f(5)$.

63. The function is not continuous at $x=5$ because $g(5)$ does not exist.

$$g(5)=\frac{1}{(5)^2-7(5)+10}=\frac{1}{25-35+10}=\frac{1}{0}.$$

64. The function $f(x)$ is continuous at $x=3$, because:
1) $f(3)$ exists,
2) $\lim_{x\to 3} f(x)$ exists, and
3) $\lim_{x\to 3} f(x)=-1=f(3)$.

65. The function is not continuous at $x=2$, because $G(2)$ does not exist.

$$G(2)=\frac{1}{(2)^2-6(2)+8}=\frac{1}{0}$$

66. The function $F(x)$ is continuous at $x=4$, because:
1) $F(4)$ exists, $\mathrm{F}(4)=-\frac{1}{2}$
2) $\lim_{x\to 4} F(x)$ exists, $\lim_{x\to 4} F(x)=-\frac{1}{2}$, and
3) $\lim_{x\to 4} F(x)=-\frac{1}{2}=F(4)$.

67. Yes, the function is continuous over the interval $(-4,4)$. Since the function is defined for every value in the interval, the Theorem on Limits of Rational Functions tells us $\lim_{x\to a} g(x)=g(a)$ for all values a in the interval. Thus $g(x)$ is continuous over the interval.

68. Yes, the function is continuous over the interval $(-5,5)$. Since the function is defined for every value in the interval, the Theorem on Limits of Rational Functions tells us $\lim_{x\to a} F(x)=F(a)$ for all values a in the interval. Thus $F(x)$ is continuous over the interval.

69. No, the function is not continuous over the interval $(0,\infty)$ because the function does not exist at $x=1$. $G(1)=\frac{1}{1-1}=\frac{1}{0}$, which is undefined.

70. No, the function is not continuous over the interval $(-7,7)$ because the function does not exist at $x=0$. $f(0)=\frac{1}{0}+3$, which is undefined.

71. Yes, the function is continuous on $\mathbb{R}$. The function is defined for all real numbers, so by the Theorem on Limits of Rational Functions, $\lim_{x\to a} g(x)=g(a)$ for all a in $\mathbb{R}$.

72. No, the function is not continuous over $\mathbb{R}$, because the function does not exist at $x=5$.
$F(5)=\frac{3}{x-5}=\frac{3}{0}$, which is undefined.

73. The limit as x approaches 20 from the left is found in this case using Limit Property L3 by substituting on the piece of the function that is defined for values less than 20. The limit from the left of the function is:
$\lim_{x\to 20^-}(1.5x)=1.50(20)=30.$
The limit as x approaches 20 from the right is found in this case using Limit Property L3 by substituting on the piece of the function that is defined for values greater than 20. The limit from the right of the function is:
$\lim_{x\to 20^+}(1.25x)=1.25(20)=25.$
Since the limit from the left, 30, is not the same as the limit from the right, 25, the limit does not exist. $\lim_{x\to 20} p(x)$ *does not exist*.

74. The limit as x approaches 100 from the left is found in this case using Limit Property L3 by substituting on the piece of the function that is defined for values less than 100. The limit from the left of the function is:
$\lim_{x\to 100^-}(0.08x)=0.08(100)=8.$
The limit as x approaches 100 from the right is found in this case using Limit Property L3 by substituting on the piece of the function that is defined for values greater than 100.
The solution is continued on the next page.

The limit from the right of the function is:
$\lim_{x\to 100^+}(0.06x) = 0.06(100) = 6.$
Since the limit from the left, 8, is not the same as the limit from the right, 6, the limit does not exist. $\lim_{x\to 100} p(x)$ *does not exist.*

75. The limit as t approaches 60 from the left is found in this case using Limit Property L3 by substituting on the piece of the function that is defined for values less than 60. The limit from the left of the function is:
$\lim_{t\to 60^-}(2t) = 2(60) = 120.$
The limit as t approaches 20 from the right is found in this case using Limit Property L3 by substituting on the piece of the function that is defined for values greater than 20. The limit from the right of the function is:
$\lim_{t\to 60^+}(300-3t) = 300-3(60) = 120.$
Since the limit from the left, 120, is the same as the limit from the right, 120 , the limit is $\lim_{t\to 60} T(t) = 120.$

76. In order for the function to be continuous at $x = 20$ the limit as x approaches 20 from the left must equal the limit as x approaches 20 from the right. The limit from the left of the function is:
$\lim_{x\to 20^-}(1.50x) = 1.50(20) = 30$. Therefore, the limit of $p(x)$ as x approaches 20 from the right must be equal to 30. We set the right hand limit equal to 30:
$\lim_{x\to 20^+}(1.25x+k) = 30.$
We allow x to approach 20. By Limit Property L3, we have:
$1.25(20)+k = 30$
Solving this equation for k yields:
$25+k = 30$
$k = 5.$
Therefore, k must equal 5 in order for the price function to be continuous at $x = 20$.

77. In exercise 74, we found that $\lim_{x\to 100^-} = 0.08(100) = 8$. In order for $p(x)$ to be continuous, the limit from the right must equal 8. Therefore,
$\lim_{x\to 100^+} 0.06x + k = 8$
$0.06(100)+k = 8$ By Limit Property L3
$6+k = 8$
$k = 2$
The constant k must equal 2 in order for the function to be continuous at $x = 100$.

78. a) As the inputs x approach 0 from the left, the outputs approach -1 . We see this by looking at a table:

x	-0.1	-0.01	-0.001
$\frac{\lvert x\rvert}{x}$	-1	-1	-1

$\lim_{x\to 0^-}\frac{|x|}{x} = -1$.
As the inputs x approach 0 from the right, the outputs approach 1 . We see this by looking at a table:

x	0.001	0.01	0.1
$\frac{\lvert x\rvert}{x}$	1	1	1

$\lim_{x\to 0^+}\frac{|x|}{x} = 1.$
Since the limit from the left, -1 , is not the same as the limit from the right, 1 , the limit does not exist. $\lim_{x\to 0}\frac{|x|}{x}$ does not exist.

b) The limit as x approaches -2 from the left is:
$$\lim_{x\to -2^-}\frac{x^3+8}{x^2-4} = -3.$$
The limit as x approaches -2 from the right is:
$$\lim_{x\to -2^+}\frac{x^3+8}{x^2-4} = -3.$$
Since the limit from the left is the same as the limit from the right, the limit exists and is:
$$\lim_{x\to -2}\frac{x^3+8}{x^2-4} = -3 .$$

The solution is continued on the next page.

Another approach to finding the limit on the previous page would be to simplify the function, by factoring the numerator and denominator and canceling common factors.

$$\begin{aligned}\lim_{x\to -2}\frac{x^3+8}{x^2-4} &= \lim_{x\to -2}\frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)}\\ &= \lim_{x\to -2}\frac{x^2-2x+4}{x-2} \quad \text{assuming } x \neq -2\\ &= \frac{(-2)^2-2(-2)+4}{(-2)-2}\\ &= \frac{12}{-4}\\ &= -3.\end{aligned}$$

79. 0.5 , or $\frac{1}{2}$

80. 6

81. 0.5, or $\frac{1}{2}$

82. −0.2887, or $-\frac{1}{2\sqrt{3}}$

83. 0.378, or $\frac{1}{\sqrt{7}}$

84. 0.75, or $\frac{3}{4}$

85. 0

86. 0.25, or $\frac{1}{4}$

Exercise Set 1.3

1. a) $f(x)=5x^2$

so,

$f(x+h)=5(x+h)^2$ substituting $x+h$ for x

$=5(x^2+2xh+h^2)$

$=5x^2+10xh+5h^2$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{(5x^2+10xh+5h^2)-5x^2}{h}$ Substituting

$=\frac{10xh+5h^2}{h}$

$=\frac{h(10x+5h)}{h}$ Factoring the numerator

$=\frac{h}{h}\cdot(10x+5h)$ Removing a factor = 1.

$=10x+5h,$ Simplified difference quotient

or $5(2x+h)$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$10x+5h$ Simplified difference quotient

$=10(5)+5(2)=60$

substituting 5 for x and 2 for h;

$=10(5)+5(1)=55$

substituting 5 for x and 1 for h;

$=10(5)+5(0.1)=50.5$

substituting 5 for x and 0.1 for h;

$=10(5)+5(0.01)=50.05$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	60
5	1	55
5	0.1	50.5
5	0.01	50.05

2. a) $f(x)=4x^2$

so,

$\frac{f(x+h)-f(x)}{h}=\frac{4(x+h)^2-4x^2}{h}$

$=\frac{(4x^2+8xh+4h^2)-4x^2}{h}$

$=\frac{8xh+4h^2}{h}$

$=\frac{h(8x+4h)}{h}$

$=\frac{h}{h}\cdot(8x+4h)$

$=8x+4h=4(2x+h)$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$8x+4h$ Simplified difference quotient

$=8(5)+4(2)=48$

substituting 5 for x and 2 for h;

$=8(5)+4(1)=44$

substituting 5 for x and 1 for h;

$=8(5)+4(0.1)=40.4$

substituting 5 for x and 0.1 for h;

$=8(5)+4(0.01)=40.04$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	48
5	1	44
5	0.1	40.4
5	0.01	40.04

3. a) $f(x) = -5x^2$

so,

$f(x+h) = -5(x+h)^2$ substituting $x+h$ for x

$= -5(x^2 + 2xh + h^2)$

$= -5x^2 - 10xh - 5h^2$

Then

$\dfrac{f(x+h) - f(x)}{h}$ Difference quotient

$= \dfrac{(-5x^2 - 10xh - 5h^2) - (-4x^2)}{h}$ Substituting

$= \dfrac{-10xh - 5h^2}{h}$

$= \dfrac{h(-10x - 5h)}{h}$ Factoring the numerator

$= \dfrac{h}{h} \cdot (-10x - 5h)$ Removing a factor = 1.

$= -10x - 5h,$ Simplified difference quotient

or $-5(2x+h)$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-10x - 5h$ Simplified difference quotient

$= -10(5) - 5(2) = -60$

substituting 5 for x and 2 for h;

$= -10(5) - 5(1) = -55$

substituting 5 for x and 1 for h;

$= -10(5) - 5(0.1) = -50.5$

substituting 5 for x and 0.1 for h;

$= -10(5) - 5(0.01) = -50.05$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\dfrac{f(x+h) - f(x)}{h}$
5	2	−60
5	1	−55
5	0.1	−50.5
5	0.01	−50.05

4. a) $f(x) = -4x^2$

so,

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{-4(x+h)^2 - (-4x^2)}{h}$

$= \dfrac{(-4x^2 - 8xh - 4h^2) + 4x^2}{h}$

$= \dfrac{-8xh - 4h^2}{h}$

$= \dfrac{h(-8x - 4h)}{h}$

$= \dfrac{h}{h} \cdot (-8x - 4h)$

$= -8x - 4h = -4(2x + h)$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-8x - 4h$ Simplified difference quotient

$= -8(5) - 4(2) = -48$

substituting 5 for x and 2 for h;

$= -8(5) - 4(1) = -44$

substituting 5 for x and 1 for h;

$= -8(5) - 4(0.1) = -40.4$

substituting 5 for x and 0.1 for h;

$= -8(5) - 4(0.01) = -40.04$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\dfrac{f(x+h) - f(x)}{h}$
5	2	−48
5	1	−44
5	0.1	−40.4
5	0.01	−40.04

5. a) $f(x) = x^2 - x$

We substitute $x + h$ for x

$$f(x+h) = (x+h)^2 - (x+h)$$
$$= (x^2 + 2xh + h^2) - x - h$$
$$= x^2 + 2xh + h^2 - x - h$$

Then

$$\frac{f(x+h) - f(x)}{h} \quad \text{Difference quotient}$$
$$= \frac{(x^2 + 2xh + h^2 - x - h) - (x^2 - x)}{h}$$
$$= \frac{2xh + h^2 - h}{h}$$
$$= \frac{h(2x + h - 1)}{h} \quad \text{Factoring the numerator}$$
$$= \frac{h}{h} \cdot (2x + h - 1) \quad \text{Removing a factor} = 1.$$
$$= 2x + h - 1 \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h - 1$ Simplified difference quotient

$= 2(5) + (2) - 1 = 11$

substituting 5 for x and 2 for h;

$= 2(5) + (1) - 1 = 10$

substituting 5 for x and 1 for h;

$= 2(5) + (0.1) - 1 = 9.1$

substituting 5 for x and 0.1 for h;

$= 2(5) + (0.01) - 1 = 9.01$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h) - f(x)}{h}$
5	2	11
5	1	10
5	0.1	9.1
5	0.01	9.01

6. a) $f(x) = x^2 + x$

Then

$$\frac{f(x+h) - f(x)}{h} \quad \text{Difference quotient}$$
$$= \frac{((x+h)^2 + (x+h)) - (x^2 + x)}{h}$$
$$= \frac{(x^2 + 2xh + h^2 + x + h) - (x^2 + x)}{h}$$
$$= \frac{2xh + h^2 + h}{h}$$
$$= \frac{h(2x + h + 1)}{h}$$
$$= \frac{h}{h} \cdot (2x + h + 1)$$
$$= 2x + h + 1 \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h + 1$ Simplified difference quotient

$= 2(5) + (2) + 1 = 13$

substituting 5 for x and 2 for h;

$= 2(5) + (1) + 1 = 12$

substituting 5 for x and 1 for h;

$= 2(5) + (0.1) + 1 = 11.1$

substituting 5 for x and 0.1 for h;

$= 2(5) + (0.01) + 1 = 11.01$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h) - f(x)}{h}$
5	2	13
5	1	12
5	0.1	11.1
5	0.01	11.01

7. a) $f(x)=\frac{9}{x}$

We substitute $x+h$ for x

$$f(x+h)=\frac{9}{x+h}$$

Then

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$=\frac{\left(\frac{9}{x+h}\right)-\left(\frac{9}{x}\right)}{h}$$

$$=\frac{\left(\frac{9}{x+h}\cdot\frac{x}{x}\right)-\left(\frac{9}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h} \quad \text{multiplying by 1}$$

$$=\frac{\left(\frac{9x}{x(x+h)}\right)-\left(\frac{9(x+h)}{x(x+h)}\right)}{h}$$

$$=\frac{\frac{-9h}{x(x+h)}}{h} \quad \text{adding fractions}$$

$$=\frac{\frac{-9h}{x(x+h)}}{\frac{h}{1}} \quad h=\frac{h}{1}$$

$$=\frac{-9h}{x(x+h)}\cdot\frac{1}{h} \quad \text{multiplying by the reciprocal}$$

$$=\frac{h}{h}\cdot\left(\frac{-9}{x(x+h)}\right) \quad \text{Removing a factor = 1.}$$

$$=\frac{-9}{x(x+h)} \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$$-\frac{9}{x(x+h)} \quad \text{Simplified difference quotient}$$

$$=-\frac{9}{5(5+2)}=-\frac{9}{35}$$

substituting 5 for x and 2 for h;

$$=-\frac{9}{5(5+1)}=-\frac{9}{30}=-\frac{3}{10}$$

substituting 5 for x and 1 for h;

$$=-\frac{9}{5(5+0.1)}=-\frac{9}{25.5}=-\frac{6}{17}$$

substituting 5 for x and 0.1 for h;

$$=-\frac{9}{5(5+0.01)}=-\frac{9}{25.05}=-\frac{60}{167}$$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	$-\frac{9}{35}$
5	1	$-\frac{3}{10}$
5	0.1	$-\frac{6}{17}$
5	0.01	$-\frac{60}{167}$

8. a) $f(x)=\frac{2}{x}$

Then

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$=\frac{\left(\frac{2}{x+h}\right)-\left(\frac{2}{x}\right)}{h}$$

$$=\frac{\left(\frac{2}{x+h}\cdot\frac{x}{x}\right)-\left(\frac{2}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h}$$

$$=\frac{\left(\frac{2x}{x(x+h)}\right)-\left(\frac{2(x+h)}{x(x+h)}\right)}{h}$$

$$=\frac{\frac{-2h}{x(x+h)}}{h}$$

$$=\frac{-2h}{x(x+h)}\cdot\frac{1}{h}$$

$$=\frac{h}{h}\cdot\left(\frac{-2}{x(x+h)}\right)$$

$$=\frac{-2}{x(x+h)} \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-\frac{2}{x(x+h)}$ Simplified difference quotient

$=-\frac{2}{5(5+2)}=-\frac{2}{35}$

substituting 5 for x and 2 for h;

$=-\frac{2}{5(5+1)}=-\frac{2}{30}=-\frac{1}{15}$

substituting 5 for x and 1 for h;

$=-\frac{2}{5(5+0.1)}=-\frac{2}{25.5}=-\frac{4}{51}$

substituting 5 for x and 0.1 for h;

$=-\frac{2}{5(5+0.01)}=-\frac{2}{25.05}=-\frac{40}{501}$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	$-\frac{2}{35}$
5	1	$-\frac{1}{15}$
5	0.1	$-\frac{4}{51}$
5	0.01	$-\frac{40}{501}$

9. a) $f(x)=2x+3$

We substitute $x+h$ for x

$f(x+h)=2(x+h)+3$

$=2x+2h+3$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{(2x+2h+3)-(2x+3)}{h}$

$=\frac{2h}{h}$

$=2$ Simplified difference quotient

b) The difference quotient is 2 for all values of x and h. Therefore, the completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	2
5	1	2
5	0.1	2
5	0.01	2

10. a) $f(x)=-2x+5$

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{(-2x-2h+5)-(-2x+5)}{h}$

$=\frac{-2h}{h}$

$=-2$ Simplified difference quotient

b) The difference quotient is -2 for all values of x and h. Therefore, the completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	-2
5	1	-2
5	0.1	-2
5	0.01	-2

11. a) $f(x)=12x^3$

so,

$f(x+h)=12(x+h)^3$ substituting $x+h$ for x

$=12(x^3+3x^2h+3xh^2+h^3)$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{12(x^3+3x^2h+3xh^2+h^3)-(12x^3)}{h}$

$=\frac{36x^2h+36xh^2+12h^3}{h}$

$=\frac{h(36x^2+36xh+12h^2)}{h}$

Factoring the numerator.

$=\frac{h}{h}\cdot(36x^2+36xh+12h^2)$

Removing a factor = 1.

$=36x^2+36xh+12h^2$

Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$36x^2+36xh+12h^2$

$=36(5)^2+36(5)(2)+12(2)^2=1308$

substituting 5 for x and 2 for h;

$=36(5)^2+36(5)(1)+12(1)^2=1092$

substituting 5 for x and 1 for h;

$=36(5)^2+36(5)(0.1)+12(0.1)^2=918.12$

substituting 5 for x and 0.1 for h;

$=36(5)^2+36(5)(0.01)+12(0.01)^2=901.8012$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	1308
5	1	1092
5	0.1	918.12
5	0.01	901.8012

12. a) $f(x)=1-x^3$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{(1-x^3-3x^2h-3xh^2-h^3)-(1-x^3)}{h}$

$=\frac{-3x^2h-3xh^2-h^3}{h}$

$=\frac{h(-3x^2-3xh-h^2)}{h}$

$=\frac{h}{h}\cdot(-3x^2-3xh-h^2)$

$=-3x^2-3xh-h^2$

Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-3x^2-3xh-h^2$

$=-3(5)^2-3(5)(2)-(2)^2=-109$

substitiuting 5 for x and 2 for h;

$=-3(5)^2-3(5)(1)-(1)^2=-91$

substitiuting 5 for x and 1 for h;

$=-3(5)^2-3(5)(0.1)-(0.1)^2=-76.51$

substitiuting 5 for x and 0.1 for h;

$=-3(5)^2-3(5)(0.01)-(0.01)^2=-75.1501$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	−109
5	1	−91
5	0.1	−76.51
5	0.01	−75.1501

13. a) $f(x)=x^2-4x$

We substitute $x+h$ for x

$f(x+h)=(x+h)^2-4(x+h)$

$=(x^2+2xh+h^2)-4x-4h$

$=x^2+2xh+h^2-4x-4h$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{((x+h)^2-4(x+h))-(x^2-4x)}{h}$

$=\frac{(x^2+2xh+h^2-4x-4h)-(x^2-4x)}{h}$

$=\frac{2xh+h^2-4h}{h}$

$=\frac{h(2x+h-4)}{h}$ Factoring the numerator.

$=\frac{h}{h}\cdot(2x+h-4)$ Removing a factor = 1.

$=2x+h-4$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x+h-4$ Simplified difference quotient

$=2(5)+(2)-4=8$

substituting 5 for x and 2 for h;

$=2(5)+(1)-4=7$

substituting 5 for x and 1 for h;

$=2(5)+(0.1)-4=6.1$

substituting 5 for x and 0.1 for h;

The solution is continued on the next page.

Continuing from the previous page:

$= 2(5) + (0.01) - 4 = 6.01$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	8
5	1	7
5	0.1	6.1
5	0.01	6.01

14. a) $f(x) = x^2 - 3x$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$= \frac{(x^2 + 2xh + h^2 - 3x - 3h) - (x^2 - 3x)}{h}$

$= \frac{2xh + h^2 - 3h}{h}$

$= \frac{h(2x + h - 3)}{h}$

$= \frac{h}{h} \cdot (2x + h - 3)$

$= 2x + h - 3$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h - 3$ Simplified difference quotient

$= 2(5) + (2) - 3 = 9$

substituting 5 for x and 2 for h;

$= 2(5) + (1) - 3 = 8$

substituting 5 for x and 1 for h;

$= 2(5) + (0.1) - 3 = 7.1$

substituting 5 for x and 0.1 for h;

$= 2(5) + (0.01) - 3 = 7.01$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	9
5	1	8
5	0.1	7.1
5	0.01	7.01

15. a) $f(x) = x^2 - 3x + 5$

We substitute $x + h$ for x

$f(x+h) = (x+h)^2 - 3(x+h) - 5$

$= (x^2 + 2xh + h^2) - 3x - 3h + 5$

$= x^2 + 2xh + h^2 - 3x - 3h + 5$

We substitute to find the simplified difference quotient.

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$= \frac{(x^2 + 2xh + h^2 - 3x - 3h + 5) - (x^2 - 3x + 5)}{h}$

$= \frac{2xh + h^2 - 3h}{h}$

$= \frac{h(2x + h - 3)}{h}$ Factoring the numerator.

$= \frac{h}{h} \cdot (2x + h - 3)$ Removing a factor = 1.

$= 2x + h - 3$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h - 3$ Simplified difference quotient

$= 2(5) + (2) - 3 = 9$

substituting 5 for x and 2 for h;

$= 2(5) + (1) - 3 = 8$

substituting 5 for x and 1 for h;

$= 2(5) + (0.1) - 3 = 7.1$

substituting 5 for x and 0.1 for h;

$= 2(5) + (0.01) - 3 = 7.01$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	9
5	1	8
5	0.1	7.1
5	0.01	7.01

16. a) $f(x)=x^2+4x-3$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{(x^2+2xh+h^2+4x+4h-3)-(x^2+4x-3)}{h}$

$=\frac{2xh+h^2+4h}{h}$

$=\frac{h(2x+h+4)}{h}$

$=\frac{h}{h}\cdot(2x+h+4)$

$=2x+h+4$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x+h+4$ Simplified difference quotient

$=2(5)+(2)+4=16$

substituting 5 for x and 2 for h;

$=2(5)+(1)+4=15$

substituting 5 for x and 1 for h;

$=2(5)+(0.1)+4=14.1$

substituting 5 for x and 0.1 for h;

$=2(5)+(0.01)+4=14.01$

substituting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	16
5	1	15
5	0.1	14.1
5	0.01	14.01

17. To find the average rate of change from 2003 to 2007, we locate the corresponding points $(2003,0)$ and $(2007,1.1)$. Using these two points we calculate the average rate of change

$\frac{1.1-0}{2007-2003}=\frac{1.1}{4}=0.275\approx 0.28.$

The average rate of change of total employment from 2003 to 2007 increased approximately 0.28% per year.

To find the average rate of change from 2007 to 2012, we locate the corresponding points $(2007,1.1)$ and $(2012,1.9)$.

Using the two points on the previous column we calculate the average rate of change

$\frac{1.9-1.1}{2012-2007}=\frac{0.8}{5}=0.16.$

The average rate of change of total employment from 2007 to 2012 increased approximately 0.16% per year.

To find the average rate of change from 2003 to 2012, we locate the corresponding points $(2003,0)$ and $(2012,1.9)$. Using these two points we calculate the average rate of change

$\frac{1.9-0}{2012-2003}=\frac{1.9}{9}=0.211\overline{1}\approx 0.21.$

The average rate of change of total employment from 2003 to 2012 increased approximately 0.21% per year.

18. Locate the points $(2003,0)$ and $(2007,1.6)$.

The average rate of change is

$\frac{1.6-0}{2007-2003}=\frac{1.6}{4}=0.4.$

The average rate of change of construction employment from 2003 to 2007 was approximately 0.4% per year.

Locate the points $(2007,1.6)$ and $(2012,5)$.

The average rate of change is

$\frac{5-1.6}{2012-2007}=\frac{3.4}{5}=0.68.$

The average rate of change of construction employment from 2007 to 2012 was approximately 0.68% per year.

Locate the points $(2003,0)$ and $(2012,5)$.

The average rate of change is

$\frac{5-0}{2012-2003}=\frac{5}{9}\approx 0.56.$

The average rate of change of construction employment from 2003 to 2012 was approximately 0.56% per year.

19. To find the average rate of change from 2003 to 2007, we locate the corresponding points $(2003,0)$ and $(2007,1.4)$. Using these two points we calculate the average rate of change

$\frac{1.4-0}{2007-2003}=\frac{1.4}{4}=0.35.$

The average rate of change of professional services occupations employment from 2003 to 2007 increased approximately 0.35% per year.

The solution is continued on the next page.

To find the average rate of change from 2007 to 2012, we locate the corresponding points $(2007, 1.4)$ and $(2012, 2.7)$. Using these two points we calculate the average rate of change

$$\frac{2.7-1.4}{2012-2007}=\frac{1.3}{5}=0.26.$$

The average rate of change of professional services occupations employment from 2007 to 2012 increased approximately 0.16% per year.

To find the average rate of change from 2003 to 2012, we locate the corresponding points $(2003, 0)$ and $(2012, 2.7)$. Using these two points we calculate the average rate of change

$$\frac{2.7-0}{2012-2003}=\frac{2.7}{9}=0.3.$$

The average rate of change of professional services occupations employment from 2003 to 2012 increased approximately 0.3% per year.

20. Locate the points $(2003, 0)$ and $(2007, 0.3)$.

The average rate of change is

$$\frac{0.3-0}{2007-2003}=\frac{0.3}{4}=0.075\approx 0.8.$$

The average rate of change of health care employment from 2003 to 2007 was approximately 0.08% per year.

Locate the points $(2007, 0.3)$ and $(2012, 0.8)$.

The average rate of change is

$$\frac{0.8-0.3}{2012-2007}=\frac{0.5}{5}=0.1.$$

The average rate of change of health care employment from 2007 to 2012 was approximately 0.1% per year.

Locate the points $(2003, 0)$ and $(2012, 0.8)$.

The average rate of change is

$$\frac{0.8-0}{2012-2003}=\frac{0.8}{9}=0.08\overline{8}\approx 0.09.$$

The average rate of change of health care employment from 2003 to 20012 was approximately 0.09% per year.

21. To find the average rate of change from 2003 to 2007, we locate the corresponding points $(2003, 0)$ and $(2007, -0.1)$. Using these two points we calculate the average rate of change

$$\frac{0-(-0.1)}{2007-2003}=\frac{-0.1}{4}=-0.025\approx -0.03.$$

The average rate of change of education employment from 2003 to 2007 increased approximately -0.03% per year.

To find the average rate of change from 2007 to 2012, we locate the corresponding points $(2007, -0.1)$ and $(2012, -0.7)$. Using these two points we calculate the average rate of change

$$\frac{-0.7-(-0.1)}{2012-2007}=\frac{-0.6}{5}=-0.12.$$

The average rate of change of education employment from 2007 to 2012 increased approximately -0.12% per year.

To find the average rate of change from 2003 to 2012, we locate the corresponding points $(2003, 0)$ and $(2012, -0.7)$. Using these two points we calculate the average rate of change

$$\frac{-0.7-0}{2012-2003}=\frac{-0.7}{9}=-0.07\overline{7}\approx -0.08.$$

The average rate of change of education employment from 2003 to 2012 increased approximately -0.08% per year.

22. Locate the points $(2003, 0)$ and $(2007, 0.3)$.

The average rate of change is

$$\frac{0.3-0}{2007-2003}=\frac{0.3}{4}=0.075\approx 0.08.$$

The average rate of change of government employment from 2003 to 2007 was approximately 0.08% per year.

Locate the points $(2007, 0.3)$ and $(2012, -1.7)$.

The average rate of change is

$$\frac{-1.7-0.3}{2012-2007}=\frac{-2}{5}=-0.4.$$

The average rate of change of government employment from 2007 to 2012 was approximately -0.4% per year.

Locate the points $(2003, 0)$ and $(2012, -1.7)$.

The average rate of change is

$$\frac{-1.7-0}{2012-2003}=\frac{-1.7}{9}=-0.18\overline{8}\approx -0.19.$$

The average rate of change of government employment from 2003 to 20012 was approximately -0.19% per year.

23. To find the average rate of change from 2003 to 2007, we locate the corresponding points $(2003, 0)$ and $(2007, 0.2)$. Using these two points we calculate the average rate of change

$$\frac{0.2-0}{2007-2003}=\frac{0.2}{4}=0.05.$$

The average rate of change of mining and logging employment from 2003 to 2007 increased approximately 0.05% per year.

To find the average rate of change from 2007 to 2012, we locate the corresponding points $(2007, 0.2)$ and $(2012, 3.8)$. Using these two points we calculate the average rate of change

$$\frac{3.8-0.2}{2012-2007}=\frac{3.6}{5}=0.72.$$

The average rate of change of mining and logging employment from 2007 to 2012 increased approximately 0.72% per year.

To find the average rate of change from 2003 to 2012, we locate the corresponding points $(2003, 0)$ and $(2012, 3.8)$. Using these two points we calculate the average rate of change

$$\frac{3.8-0}{2012-2003}=\frac{3.8}{9}=0.422\overline{2}\approx 0.42$$

The average rate of change of mining and logging employment from 2003 to 2012 increased approximately 0.42% per year.

24. Locate the points $(2003, 0)$ and $(2007, -0.8)$.

The average rate of change is

$$\frac{-0.8-0}{2007-2003}=\frac{-0.8}{4}=-0.2.$$

The average rate of change of manufacturing employment from 2003 to 2007 was approximately -0.2% per year.

Locate the points $(2007, -0.8)$ and $(2012, 2.7)$.

The average rate of change is

$$\frac{2.7-(-0.8)}{2012-2007}=\frac{3.5}{5}=0.7.$$

The average rate of change of manufacturing employment from 2007 to 2012 was approximately 0.7% per year.

Locate the points $(2003, 0)$ and $(2012, 2.7)$. The average rate of change is

$$\frac{2.7-0}{2012-2003}=\frac{2.7}{9}=0.3.$$

The average rate of change of manufacturing employment from 2003 to 20012 was approximately 0.3% per year.

25. In order to find the average rate of change from 1982 to 1992, we use the data points $(1982, 73.10)$ and $(1992, 85.78)$. The average rate of change is

$$\frac{85.78-73.10}{1992-1982}=\frac{12.68}{10}=1.268\ .$$

Between the years 1982 to 1992 the average rate of change in U.S. energy consumption was about 1.268 quadrillion BTUs per year.

In order to find the average rate of change from 1992 to 2002, we use the data points $(1992, 85.78)$ and $(2002, 97.65)$. The average rate of change is

$$\frac{97.65-85.78}{2002-1992}=\frac{11.87}{10}=1.187\ .$$

Between the years 1992 to 2002 the average rate of change in U.S. energy consumption was about 1.187 quadrillion BTUs per year.

In order to find the average rate of change from 2002 to 2012, we use the data points $(2002, 97.65)$ and $(2012, 95.10)$. The average rate of change is

$$\frac{95.10-97.65}{2012-2002}=\frac{-2.55}{10}=-0.255\ .$$

Between the years 2002 to 2012 the average rate of change in U.S. energy consumption was about -0.225 quadrillion BTUs per year.

26. Using the points $(2007, 84.3)$ and $(2009, 44.7)$ the average rate of change is

$$\frac{44.7-84.3}{2009-2007}=\frac{-39.6}{2}=-19.8$$

Between 2007 and 2009, the average rate of change of the U.S. trade deficit with Japan was -19.8 billion dollars per year.

Using the points $(2009, 44.7)$ and $(2011, 63.2)$ the average rate of change is

$$\frac{63.2-44.7}{2011-2009}=\frac{18.5}{2}=9.25$$

Between 2009 and 2011, the average rate of change of the U.S. trade deficit with Japan was 9.25 billion dollars per year.

The solution is continued on the next page.

Using the points $(2011, 63.2)$ and $(2013, 61.6)$ the average rate of change is

$$\frac{61.6-63.2}{2013-2011}=\frac{-1.6}{2}=-0.8$$

Between 2011 and 2013, the average rate of change of the U.S. trade deficit with Japan was -0.8 billion dollars per year.

27. a) From 0 units to 1 unit the average rate of change is

$$\frac{70-0}{1-0}=70 \text{ pleasure units per unit.}$$

From 1 unit to 2 units the average rate of change is

$$\frac{109-70}{2-1}=\frac{39}{1}=39 \text{ pleasure units per unit.}$$

From 2 units to 3 units the average rate of change is

$$\frac{138-109}{3-2}=\frac{29}{1}=29 \text{ pleasure units per unit.}$$

From 3 units to 4 units the average rate of change is

$$\frac{161-138}{4-3}=\frac{23}{1}=23 \text{ pleasure units per unit.}$$

b) ✎ Answers will vary. As you consume more and more of a good, the additional utility, or amount of pleasure, associated with that good will start to fall. The additional satisfaction from the 1st to the 2nd slice of pizza is greater than the additional satisfaction from the 9th to the 10th slice of pizza.

28. a) $N(0)=0,\ N(1)=300$

$$\frac{300-0}{1-0}=300 \text{ units per thousand dollars.}$$

$N(1)=300,\ N(2)=480$

$$\frac{480-300}{2-1}=180 \text{ units per thousand dollars.}$$

$N(2)=480,\ N(3)=600$

$$\frac{600-480}{3-2}=120 \text{ units per thousand dollars.}$$

$N(3)=600,\ N(4)=700$

$$\frac{700-600}{4-3}=100 \text{ units per thousand dollars.}$$

b) ✎ As spending on advertising increases, there are few increases in sales from each additional amount of spending.

29. $p(x)=0.06x^3-0.5x^2+1.64x+24.76$

a) $p(4)=0.06(4)^3-0.5(4)^2+1.64(4)+24.76$

$p(4)=27.16$

b) $p(6)=0.06(6)^3-0.5(6)^2+1.64(6)+24.76$

$p(6)=29.56$

c) $P(6)-p(4)=29.56-27.16=2.40$

d) $\dfrac{p(6)-p(4)}{6-4}=\dfrac{2.40}{2}=1.20$

This result implies that the average price of a ticket between 2012 (x =4) and 2014 (x =6) grew at an average rate of \$1.20 per year.

30. $A(t)=2000(1.015)^{4t}$

a) $A(3)=2000(1.015)^{4(3)}\approx 2391.24$

b) $A(5)=2000(1.015)^{4(5)}\approx 2693.71$

c) $A(5)-A(3)=2693.710013-2391.236343$
$=302.47$

d) $\dfrac{A(5)-A(3)}{5-3}=\dfrac{302.4736702}{2}=151.236835$

The value of the account grew at a rate of \$151.24 per year between the 3rd and 5th year of the investment.

31. $P(t)=5400(0.975)^t$

$P(8)=5400(0.975)^8=4409.919\approx 4410$

$P(5)=5400(0.975)^5=4757.916\approx 4758$

$$\frac{P(8)-P(5)}{8-5}=\frac{4410-4758}{8-5}=-116$$

The population of Payton county lost on average 116 people per year between the 5th and 8th years after the last census.

32. $P(t)=17,000(1.042)^t$

$P(6)=17,000(1.042)^6=21,759.82$

$P(2)=17,000(1.042)^2=18,457.99$

$$\frac{P(6)-P(2)}{6-2}=\frac{21,759.82-18,457.99}{6-2}=825.46$$

The undergraduate population of Harbor College was rising on average at a rate of 825.46 students per year between the 2nd and 6th years.

33. $C(x) = -0.05x^2 + 50x$

First substitute 305 for *x*.

$$\begin{aligned} C(305) &= -0.05(305)^2 + 50(305) \\ &= -4651.25 + 15{,}250 \\ &= 10{,}598.75 \end{aligned}$$

The total cost of producing 305 units is \$10,598.75.

Next substitute 300 for *x*.

$$\begin{aligned} C(300) &= -0.05(300)^2 + 50(300) \\ &= -4500 + 15{,}000 \\ &= 10{,}500.00 \end{aligned}$$

The total cost of producing 300 units is \$10,500.00.

Now we can substitute to find the average rate of change.

$$\begin{aligned} \frac{C(305) - C(300)}{305 - 300} &= \frac{10{,}598.75 - 10{,}500}{305 - 300} \\ &= \frac{98.75}{5} \\ &= 19.75 \end{aligned}$$

The average cost of production between the 300th unit and 305th unit is 19.75 per unit.

34. $R(x) = -0.001x^2 + 150x$

$$\begin{aligned} R(305) &= -0.001(305)^2 + 150(305) \\ &= 45{,}656.975 \end{aligned}$$

$$\begin{aligned} R(300) &= -0.001(300)^2 + 150(300) \\ &= 45{,}910.00 \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{R(305) - R(300)}{305 - 300} &= \frac{45{,}656.975 - 45{,}910.00}{5} \\ &= 149.395 \approx 149.40 \end{aligned}$$

The average increase in revenue between the sales of the 300th unit and the 305th unit is \$149.40 per unit.

35. Note: Answers will vary according to the values estimated from the graph.

a) Locate the points $(0,8)$ and $(12,20)$ on the girls growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{20-8}{12-0} = \frac{12}{12} = 1 .$$

The average growth rate of a girl during her first 12 months is 1 pound per month.

b) Locate the points $(12,20)$ and $(24,26.5)$ on the girls growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{26.5-20}{24-12} = \frac{6.5}{12} = 0.541\overline{6} \approx 0.54 .$$

The average growth rate of a girl during her second 12 months is approximately 0.54 pounds per month.

c) Locate the points $(0,8)$ and $(24,26.5)$ on the girls growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{26.5-8}{24-0} = \frac{18.5}{24} = 0.7708\overline{3} \approx 0.77 .$$

The average growth rate of a girl during her first 24 months is approximately 0.77 pounds per month.

d) We estimate the growth rate of a 12 month old girl to be approximately 0.67 pounds per month. This answer will vary depending upon your tangent line.

e) The graph indicates that the growth rate is fastest during the first 3 months.

36. Note: Answers will vary according to the values estimated from the graph.

a) Locate the points $(0,8)$ and $(15,25)$ on the boys growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{25-8}{15-0} = \frac{17}{15} = 1.13\overline{3} \approx 1.13 .$$

The average growth rate of a boy during his first 15 months is 1.1 pound per month.

b) Locate the points $(15,25)$ and $(30,30)$ on the boys growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{30-25}{30-15} = \frac{5}{15} = 0.33\overline{3} \approx 0.3 .$$

The average growth rate of a boy during his second 15 months is approximately 0.3 pounds per month.

c) Locate the points $(0,8)$ and $(30,30)$ on the boys growth median weight chart. Using these points we calculate the average growth rate.

$\frac{30-8}{30-0}=\frac{22}{30}=0.733\overline{3}\approx 0.7$.

The average growth rate of a boy during his first 30 months is approximately 0.7 pounds per month.

d) We estimate the growth rate of a 15 month old boy to be approximately 0.5 pounds per month. We used a straight edge to draw a line tangent to the curve at the point $(15,25)$ and found the slope of that line. This answer will vary depending upon your tangent line.

37. $H(w)=0.11w^{1.36}$

a) First we substitute 500 and 700 in for w to find the home range at the respective weights.

$$\begin{aligned}H(500)&=0.11(500)^{1.36}\\&=0.11(4683.809314)\\&=515.2190246\\&\approx 515.22\end{aligned}$$

$$\begin{aligned}H(700)&=0.11(700)^{1.36}\\&=0.11(7401.731628)\\&=814.1904791\\&\approx 814.19\end{aligned}$$

Next we use the function values to find the average rate at which the mammal's home range will increase

$$\begin{aligned}\frac{H(700)-H(500)}{700-500}&=\frac{814.19-515.22}{700-500}\\&=\frac{298.97}{200}\\&\approx 1.49485\end{aligned}$$

The average rate at which a carnivorous mammal's home range increases as the animal's weight grows from 500 g to 700 g is approximately 1.49 hectares per gram.

b) First we substitute 200 and 300 in for w to find the home range at the respective weights.

$$\begin{aligned}H(200)&=0.11(200)^{1.36}\\&=0.11(1347.102971)\\&=148.1813269\\&\approx 148.18\end{aligned}$$

$$\begin{aligned}H(300)&=0.11(300)^{1.36}\\&=0.11(2338.217499)\\&=257.2039249\\&\approx 257.20\end{aligned}$$

Next we use the function values to find the average rate at which the mammal's home range will increase

$$\begin{aligned}\frac{H(300)-H(200)}{300-200}&=\frac{257.20-148.18}{300-200}\\&=\frac{109.02}{100}\\&\approx 1.0902\end{aligned}$$

The average rate at which a carnivorous mammal's home range increases as the animal's weight grows from 200 g to 300 g is approximately 1.09 hectares per gram.

38. $R(x)=11.74x^{1/4}$

a) First, find the function values.

$R(40{,}000)=11.74(40{,}000)^{1/4}\approx 166.03$

$R(60{,}000)=11.74(60{,}000)^{1/4}\approx 183.74$

Next find the average rate of change:

$$\begin{aligned}\frac{R(60{,}000)-R(40{,}000)}{60{,}000-40{,}000}&\approx\frac{183.74-166.03}{60{,}000-40{,}000}\\&\approx 0.00089\end{aligned}$$

The rate at which the maximum radar range changes as peak power increases from 40,000 W to 60,000 W is approximately 0.00089 miles per watt.

b) First, find the function values.

$R(50{,}000)=11.74(50{,}000)^{1/4}\approx 175.55$

$R(60{,}000)=11.74(60{,}000)^{1/4}\approx 183.74$

Next find the average rate of change:

$$\begin{aligned}\frac{R(60{,}000)-R(50{,}000)}{60{,}000-50{,}000}&\approx\frac{183.74-175.55}{60{,}000-50{,}000}\\&\approx\frac{8.19}{10{,}000}\\&\approx 0.00082\end{aligned}$$

The rate at which the maximum radar range changes as peak power increases from 50,000 W to 60,000 W is approximately 0.00082 miles per watt.

39. a) We locate the points $(0,0)$ and $(8,10)$ on the graph and use them to calculate the average rate of change.

$$\frac{10-0}{8-0}=\frac{10}{8}=\frac{5}{4}=1.25$$

The average rate of change is 1.25 words per minute.

We locate the points $(8,10)$ and $(16,20)$ on the graph and use them to calculate the average rate of change.

$$\frac{20-10}{16-8}=\frac{10}{8}=\frac{5}{4}=1.25$$

The average rate of change is 1.25 words per minute.

We locate the points $(16,20)$ and $(24,25)$ on the graph and use them to calculate the average rate of change.

$$\frac{25-20}{24-16}=\frac{5}{8}=0.625$$

The average rate of change is 0.625 words per minute.

We locate the points $(24,25)$ and $(32,25)$ on the graph and use them to calculate the average rate of change.

$$\frac{25-25}{32-24}=\frac{0}{8}=0$$

The average rate of change is 0 words per minute.

We locate the points $(32,25)$ and $(36,25)$ on the graph and use them to calculate the average rate of change.

$$\frac{25-25}{36-32}=\frac{0}{4}=0$$

The average rate of change is 0 words per minute.

b) ✎ Answers will vary. The person has reached a saturation point after 24 minutes. They cannot memorize any more words.

40. a) We use the two points $(0,30680)$ and $(13.5,31077)$.

$$\frac{31,077-30,680}{13.5-0}=\frac{397}{13.5}\approx 29.4$$

The car averaged 29.4 miles per gallon.

b) To find the average rate of gas consumption in gallons per mile, we take the reciprocal of part (a).

$$\frac{13.5-0}{31,077-30,680}=\frac{13.5}{397}\approx 0.034$$

The car consumed approximately 0.034 gallons per mile on the trip.

41. $s(t)=16t^2$

a) First, we find the function values by substituting 3 and 5 in for t respectively.

$$s(3)=16(3)^2=16(9)=144$$

$$s(5)=16(5)^2=16(25)=400$$

Next we subtract the function values.

$$s(5)-s(3)=400-144=256$$

The object will fall 256 feet in the two second time period between $t=3$ and $t=5$.

b) The average rate of change is calculated as

$$\frac{s(5)-s(3)}{5-3}=\frac{400-144}{5-3}=\frac{256}{2}=128$$

The average velocity of the object during the two second time period from $t=3$ to $t=5$ is 128 feet per second.

42. $s(t)=10t^2$

a) First, find the function values.

$$s(2)=10(2)^2=40$$

$$s(5)=10(5)^5=250$$

Therefore,

$$s(5)-s(2)=250-40=210.$$

This represents the distance the truck traveled, 210 miles, in the three hour period from $t=2$ and $t=5$.

b) We have:

$$\frac{s(5)-s(2)}{5-2}=\frac{250-40}{5-2}=\frac{210}{3}=70$$

The average velocity of the truck for the three hour period between $t=2$ and $t=5$ was 70 miles per hour.

43. a) For each curve, as t changes from 0 to 4, $P(t)$ changes from 0 to 500. Thus, the average growth rate for each country is

$$\frac{500-0}{4-0}=\frac{500}{4}=125.$$

The average growth rate for each country is approximately 125 million people per year.

b) ✎ No; and average rate of change does not give an indication of the growth patterns of the two populations during the 4 years for which it was calculated.

c) For Country A:
As t changes from 0 to 1, $P(t)$ changes from 0 to 290. Thus the average growth rate is
$\frac{290-0}{1-0} = 290$ million people per year.
As t changes from 1 to 2, $P(t)$ changes from 290 to 250. Thus the average growth rate is
$\frac{250-290}{2-1} = -40$ million people per year.
As t changes from 2 to 3, $P(t)$ changes from 250 to 200. Thus the average growth rate is
$\frac{200-250}{3-2} = -50$ million people per year.
As t changes from 3 to 4, $P(t)$ changes from 200 to 500. Thus the average growth rate is
$\frac{500-200}{4-3} = 300$ million people per year.
For Country B we calculate the average growth rates at the top of the next column.
As t changes from 0 to 1, $P(t)$ changes from 0 to 125. Thus the average growth rate is
$\frac{125-0}{1-0} = 125$ million people per year.
As t changes from 1 to 2, $P(t)$ changes from 125 to 250. Thus the average growth rate is
$\frac{250-125}{2-1} = 125$ million people per year.
As t changes from 2 to 3, $P(t)$ changes from 250 to 375. Thus the average growth rate is
$\frac{375-250}{3-2} = 125$ million people per year.
As t changes from 3 to 4, $P(t)$ changes from 375 to 500. Thus the average growth rate is
$\frac{500-375}{4-3} = 125$ million people per year.

d) The statement most accurately reflects the information found in Country B. Country B shows linear growth, which implies a constant rate of change.

44. Answers will vary. The first graph shows a constant average rate of change while the second graph has average rate of change that varies on an increasing trend. The first graph shows a decreasing rate of change, while the second graph shows a varying rate of change.

45. a) Tracing along the 4-year private school graph, we see that the largest increase in costs occurred in the 2006-07 year.

b) Tracing along the 4-year public school graph, we see that the largest increases in costs occurred during the 2003-04 year and the 2008-09 year.

c) For the 4-year public school, the cost in year 2010 dollars is approximately \$10,711. To find out what the cost was in 2000 dollars assuming a 3% inflation rate over the 10 years we create the following equation using the simple compound interest formula from section R1. $C(1+r)^t = A$.

$$C(1+.03)^{10} = 10,711$$
$$C(1.03)^{10} = 10,711$$
$$C = \frac{10,711}{(1.03)^{10}}$$
$$C = 7969.989 \approx 7970$$

The cost of attending a 4 year public school in 2000 was \$7970 in year 2000 dollars. Likewise, for the 4-year private school, the cost in year 2010 dollars is approximately \$27,054.
To find out what the cost was in year 2000 dollars assuming a 3% inflation rate over the 10 years we create the following equation using the simple compound interest formula from section R1 $C(1+r)^t = A$.

$$C(1+.03)^{10} = 27,054$$
$$C(1.03)^{10} = 27,054$$
$$C = \frac{27,054}{(1.03)^{10}}$$
$$C = 20,130.71$$
$$C \approx 20,130$$

The cost of attending a 4 year private school in 2000 was \$20,130 in year 2000 dollars.

46. $f(x) = mx + b$

$$\frac{f(x+h)-f(x)}{h} = \frac{(m(x+h)+b)-(mx+b)}{h}$$
$$= \frac{mx+mh+b-mx-b}{h}$$
$$= \frac{mh}{h}$$
$$= m$$

47. $f(x) = ax^2 + bx + c$

Substituting $x + h$ for x we have,

$$f(x+h) = a(x+h)^2 + b(x+h) + c$$
$$= a\left(x^2 + 2xh + h^2\right) + bx + bh + c$$
$$= ax^2 + 2axh + ah^2 + bx + bh + c$$

Thus,

$$\frac{f(x+h) - f(x)}{h} \quad \text{Difference quotient}$$
$$= \frac{\left(ax^2 + 2axh + ah^2 + bx + bh + c\right) - \left(ax^2 + bx + c\right)}{h}$$
$$= \frac{2axh + ah^2 + bh}{h}$$
$$= \frac{h(2ax + ah + b)}{h} \quad \text{Factoring the numerator}$$
$$= 2ax + ah + b \quad \text{Simplified difference quotient}$$

48. $f(x) = ax^3 + bx^2$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{a(x+h)^3 + b(x+h)^2 - \left(ax^3 + bx^2\right)}{h}$$
$$= \frac{a\left(x^3 + 3x^2h + 3xh^2 + h^3\right) + b\left(x^2 + 2hx + h^2\right) - ax^3 - bx^2}{h}$$
$$= \frac{ax^3 + 3ax^2h + 3axh^2 + ah^3 + bx^2 + 2bhx + bh^2 - ax^3 - bx^2}{h}$$
$$= \frac{3ax^2h + 3axh^2 + ah^3 + 2bhx + bh^2}{h}$$
$$= \frac{h\left(3ax^2 + 3axh + ah^2 + 2bx + bh\right)}{h}$$
$$= 3ax^2 + 3axh + ah^2 + 2bx + bh$$

49. $f(x) = x^4$

Substituting $x + h$ for x we have,

$$f(x+h) = (x+h)^4$$
$$= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

Thus,

$$\frac{f(x+h) - f(x)}{h} \quad \text{Difference quotient}$$
$$= \frac{\left(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4\right) - \left(x^4\right)}{h}$$
$$= \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$
$$= \frac{h\left(4x^3 + 6x^2h + 4xh^2 + h^3\right)}{h} \quad \text{Factoring the numerator}$$
$$= 4x^3 + 6x^2h + 4xh^2 + h^3 \quad \text{Simplified difference quotient}$$

50. $f(x) = x^5$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{(x+h)^5 - x^5}{h}$$
$$= \frac{\left(x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5\right) - \left(x^4\right)}{h}$$
$$= \frac{5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5}{h}$$
$$= \frac{h\left(5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4\right)}{h}$$
$$= 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$$

51. $f(x) = ax^5 + bx^4$

Substituting $x+h$ for x we have,

$$f(x+h) = a(x+h)^5 + b(x+h)^4$$
$$= ax^5 + 5ax^4h + 10ax^3h^2 + 10ax^2h^3 + 5axh^4 + ah^5 + bx^4 + 4bx^3h + 6bx^2h^2 + 4bxh^3 + bh^4$$

Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$= \frac{ax^5 + 5ax^4h + 10ax^3h^2 + 10ax^2h^3 + 5axh^4 + ah^5 + bx^4 + 4bx^3h + 6bx^2h^2 + 4bxh^3 + bh^4 - (ax^5 + bx^4)}{h}$$

$$= \frac{5ax^4h + 10ax^3h^2 + 10ax^2h^3 + 5axh^4 + ah^5 + 4bx^3h + 6bx^2h^2 + 4bxh^3 + bh^4}{h}$$

$$= \frac{h(5ax^4 + 10ax^3h + 10ax^2h^2 + 5axh^3 + ah^4 + 4bx^3 + 6bx^2h + 4bxh^2 + bh^3)}{h} \quad \text{Factoring the numerator}$$

$$= 5ax^4 + 10ax^3h + 10ax^2h^2 + 5axh^3 + ah^4 + 4bx^3 + 6bx^2h + 4bxh^2 + bh^3 \quad \text{Simplified difference quotient}$$

52. $f(x) = \frac{1}{x^2}$

Substituting $x+h$ for x we have,

$f(x+h) = \frac{1}{(x+h)^2}$ Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$= \frac{\left(\frac{1}{(x+h)^2}\right) - \left(\frac{1}{x^2}\right)}{h}$$

We find a common denominator in the numerator.

$$= \frac{\left(\frac{1}{(x+h)^2}\cdot\frac{x^2}{x^2}\right) - \left(\frac{1}{x^2}\cdot\frac{(x+h)^2}{(x+h)^2}\right)}{h}$$

$$= \frac{\frac{x^2}{x^2(x+h)^2} - \frac{x^2+2xh+h^2}{x^2(x+h)^2}}{h}$$

$$= \frac{\frac{-2xh-h^2}{x^2(x+h)^2}}{h}$$

$$= \frac{-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}$$

$$= \frac{h(-2x-h)}{x^2(x+h)^2}\cdot\frac{1}{h}$$

$$= \frac{-2x-h}{x^2(x+h)^2} \quad \text{Simplified difference quotient}$$

53. $f(x) = \frac{1}{1-x}$

$$\frac{f(x+h)-f(x)}{h}$$

$$= \frac{\frac{1}{1-(x+h)} - \left(\frac{1}{1-x}\right)}{h}$$

$$= \frac{\frac{1}{1-x-h}\cdot\frac{1-x}{1-x} - \left(\frac{1}{1-x}\cdot\frac{1-x-h}{1-x-h}\right)}{h}$$

$$= \frac{\frac{1-x}{(1-x-h)(1-x)} - \frac{1-x-h}{(1-x-h)(1-x)}}{h}$$

$$= \frac{\frac{h}{(1-x-h)(1-x)}}{h}$$

$$= \frac{h}{(1-x-h)(1-x)}\cdot\frac{1}{h}$$

$$= \frac{1}{(1-x-h)(1-x)}$$

54. $f(x)=\sqrt{x}$

The difference quotient is:

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

a) $\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$ Reason: Multiplying by 1.

b) $\frac{x+h+\sqrt{x}\sqrt{x+h}-\sqrt{x}\sqrt{x+h}-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$ Reason: Expanding the numerator.

c) $\frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$ Reason: $\sqrt{x}\sqrt{x+h}-\sqrt{x}\sqrt{x+h}=0$.

d) $\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$ Reason: $x-x=0$.

e) $\frac{1}{\sqrt{x+h}+\sqrt{x}}$ Reason: $\frac{h}{h}=1$.

55. $f(x)=\sqrt{2x+1}$

Substituting $x+h$ for x we have,

$$f(x+h)=\sqrt{2(x+h)+1}$$

Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$=\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}$$

Next we rationalize the numerator and simplify.

$$=\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}\cdot\frac{\sqrt{2(x+h)+1}+\sqrt{2x+1}}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$$

$$=\frac{2(x+h)+1-(2x+1)}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}$$

$$=\frac{2x-2h+1-2x-1}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}$$

$$=\frac{-2h}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}$$

$$=\frac{-2}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$$

56. $f(x)=\frac{1}{\sqrt{x}}$

Substituting $x+h$ for x we have,

$$f(x+h)=\frac{1}{\sqrt{x+h}}$$

Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$=\frac{\left(\frac{1}{\sqrt{x+h}}\right)-\left(\frac{1}{\sqrt{x}}\right)}{h}$$

Find a common denominator in the numerator.

$$=\frac{\left(\frac{1}{\sqrt{x+h}}\cdot\frac{\sqrt{x}}{\sqrt{x}}\right)-\left(\frac{1}{\sqrt{x}}\cdot\frac{\sqrt{x+h}}{\sqrt{x+h}}\right)}{h}$$

$$=\frac{\frac{\sqrt{x}}{\sqrt{x}\sqrt{x+h}}-\frac{\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$

$$=\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$

$$=\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\cdot\frac{1}{h} \quad \text{Simplifying the complex fraction}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

Next, we rationalize the numerator on the next page.

Rationalizing the numerator, we have:

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$$

$$=\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

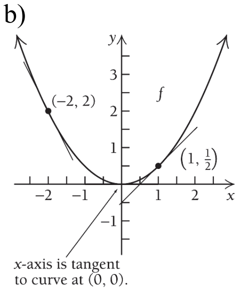
$$=\frac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

$$=\frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

Exercise Set 1.4

1. $f(x)=\frac{1}{2}x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{1}{2}(x+h)^2-\frac{1}{2}x^2}{h}$$
$$=\frac{\frac{1}{2}(x^2+2xh+h^2)-\frac{1}{2}x^2}{h}$$
$$=\frac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2-\frac{1}{2}x^2}{h}$$
$$=\frac{xh+\frac{1}{2}h^2}{h}$$
$$=\frac{h\left(x+\frac{1}{2}h\right)}{h}$$
$$=x+\frac{1}{2}h \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(x+\frac{1}{2}h\right)$$
$$=x$$

Thus, $f'(x)=x$.

d) Find the values of the derivative by making the appropriate substitutions.

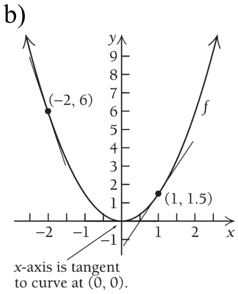
$f'(-2)=(-2)=-2$ Substituting -2 for x

$f'(0)=(0)=0$ Substituting 0 for x

$f'(1)=(1)=1$ Substituting 1 for x

2. $f(x)=\frac{3}{2}x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{3}{2}(x+h)^2-\frac{3}{2}x^2}{h}$$
$$=\frac{\frac{3}{2}(x^2+2xh+h^2)-\frac{3}{2}x^2}{h}$$
$$=\frac{\frac{3}{2}x^2+3xh+\frac{3}{2}h^2-\frac{3}{2}x^2}{h}$$
$$=\frac{3xh+\frac{3}{2}h^2}{h}$$
$$=\frac{h\left(3x+\frac{3}{2}h\right)}{h}$$
$$=3x+\frac{3}{2}h \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(3x+\frac{3}{2}h\right)$$
$$=3x$$

Thus, $f'(x)=3x$.

d) Find the values of the derivative by making the appropriate substitutions.

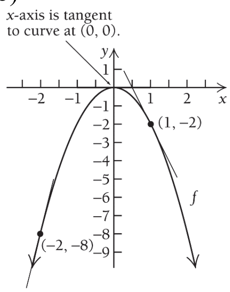
$f'(-2)=3(-2)=-6$ Substituting -2 for x

$f'(0)=3(0)=0$ Substituting 0 for x

$f'(1)=3(1)=3$ Substituting 1 for x

3. $f(x) = -2x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{-2(x+h)^2 - (-2x^2)}{h}$$
$$= \frac{-2(x^2+2xh+h^2)-(-2x^2)}{h}$$
$$= \frac{-2x^2-4xh-2h^2+2x^2}{h}$$
$$= \frac{-4xh-2h^2}{h}$$
$$= \frac{h(-4x-2h)}{h}$$
$= -4x - 2h$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-4x-2h)$$
$$= -4x$$

Thus, $f'(x) = -4x$.

d) Find the values of the derivative by making the appropriate substitutions.

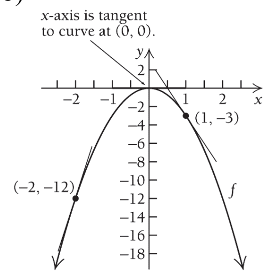
$f'(-2) = -4(-2) = 8$ Substituting -2 for x

$f'(0) = -4(0) = 0$ Substituting 0 for x

$f'(1) = -4(1) = -4$ Substituting 1 for x

4. $f(x) = -3x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{-3(x+h)^2 - (-3x^2)}{h}$$
$$= \frac{-3(x^2+2xh+h^2)-(-3x^2)}{h}$$
$$= \frac{-3x^2-6xh-3h^2+3x^2}{h}$$
$$= \frac{-6xh-3h^2}{h}$$
$$= \frac{h(-6x-3h)}{h}$$
$= -6x - 3h$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-6x-3h)$$
$$= -6x$$

Thus, $f'(x) = -6x$.

d) Find the values of the derivative by making the appropriate substitutions.

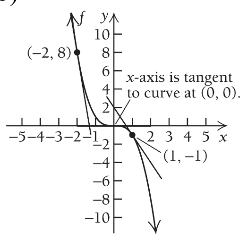
$f'(-2) = -6(-2) = 12$ Substituting -2 for x

$f'(0) = -6(0) = 0$ Substituting 0 for x

$f'(1) = -6(1) = -6$ Substituting 1 for x

5. $f(x) = -x^3$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{-(x+h)^3 - (-x^3)}{h}$$
$$= \frac{-(x^3+3x^2h+3xh^2+h^3) - (-x^3)}{h}$$
$$= \frac{-x^3-3x^2h-3xh^2-h^3+x^3}{h}$$
$$= \frac{-3x^2h-3xh^2-h^3}{h}$$
$$= \frac{h(-3x^2-3xh-h^2)}{h}$$
$$= -3x^2-3xh-h^2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-3x^2-3xh-h^2)$$
$$= -3x^2$$

Thus, $f'(x) = -3x^2$.

d) Find the values of the derivative by making the appropriate substitutions.

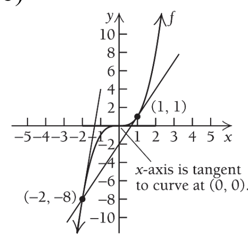
$f'(-2) = -3(-2)^2 = -12$ Substituting -2 for x

$f'(0) = -3(0)^2 = 0$ Substituting 0 for x

$f'(1) = -3(1)^2 = -3$ Substituting 1 for x

6. $f(x) = x^3$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{(x+h)^3 - (x^3)}{h}$$
$$= \frac{(x^3+3x^2h+3xh^2+h^3) - (x^3)}{h}$$
$$= \frac{3x^2h+3xh^2+h^3}{h}$$
$$= \frac{h(3x^2+3xh+h^2)}{h}$$
$$= 3x^2+3xh+h^2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(3x^2+3xh+h^2)$$
$$= 3x^2$$

Thus, $f'(x) = 3x^2$.

d) Find the values of the derivative by making the appropriate substitutions.

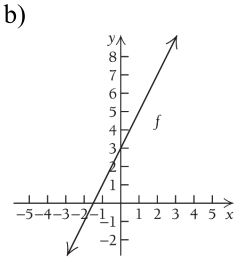
$f'(-2) = 3(-2)^2 = 12$ Substituting -2 for x

$f'(0) = 3(0)^2 = 0$ Substituting 0 for x

$f'(1) = 3(1)^2 = 3$ Substituting 1 for x

7. $f(x) = 2x + 3$

a), b)



Note: for linear functions the tangent line is the line itself.

c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{2(x+h)+3-(2x+3)}{h}$$
$$=\frac{2x+2h+3-2x-3}{h}$$
$$=\frac{2h}{h}$$
$$=2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(2)$$
$$=2$$

Thus, $f'(x)=2$.

d) Since the derivative is a constant, the value of the derivative will be 2 regardless of the value of x.

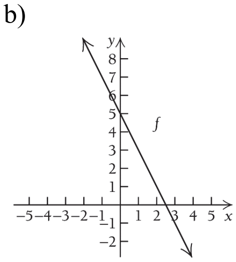
$f'(-2)=2$ Substituting -2 for x

$f'(0)=2$ Substituting 0 for x

$f'(1)=2$ Substituting 1 for x

8. $f(x)=-2x+5$

a), b)



Note: for linear functions the tangent line is the line itself.

c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{-2(x+h)+5-(-2x+5)}{h}$$
$$=\frac{-2x-2h+5+2x-5}{h}$$
$$=\frac{-2h}{h}$$
$$=-2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(-2)$$
$$=-2$$

Thus, $f'(x)=-2$.

d) Since the derivative is a constant, the value of the derivative will be -2 regardless of the value of x.

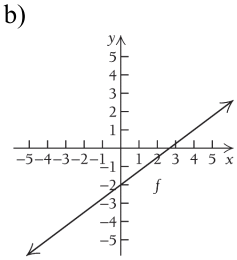
$f'(-2)=-2$ Substituting -2 for x

$f'(0)=-2$ Substituting 0 for x

$f'(1)=-2$ Substituting 1 for x

9. $f(x)=\frac{3}{4}x-2$

a), b)



Note: for linear functions the tangent line is the line itself.

c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{3}{4}(x+h)-2-\left(\frac{3}{4}x-2\right)}{h}$$
$$=\frac{\frac{3}{4}x+\frac{3}{4}h-2-\frac{3}{4}x+2}{h}$$
$$=\frac{\frac{3}{4}h}{h}$$
$$=\frac{3}{4} \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{3}{4}\right)$$
$$=\frac{3}{4}$$

Thus, $f'(x)=\frac{3}{4}$.

d) Since the derivative is a constant, the value of the derivative will be $\frac{3}{4}$ regardless of the value of x.

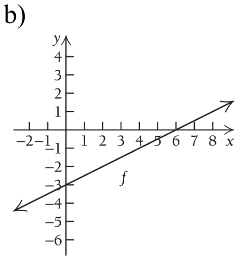
$f'(-2)=\frac{3}{4}$ Substituting -2 for x

$f'(0)=\frac{3}{4}$ Substituting 0 for x

$f'(1)=\frac{3}{4}$ Substituting 1 for x

10. $f(x)=\frac{1}{2}x-3$

a), b)



Note: for linear functions the tangent line is the line itself.

c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{1}{2}(x+h)-3-\left(\frac{1}{2}x-3\right)}{h}$$
$$=\frac{\frac{1}{2}x+\frac{1}{2}h-3-\frac{1}{2}x+3}{h}$$
$$=\frac{\frac{1}{2}h}{h}$$
$$=\frac{1}{2} \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{1}{2}\right)=\frac{1}{2}$$

Thus, $f'(x)=\frac{1}{2}$.

d) Since the derivative is a constant, the value of the derivative will be $\frac{1}{2}$ regardless of the value of x.

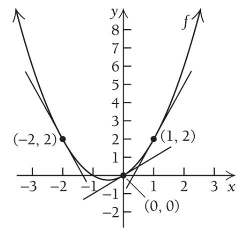
$f'(-2)=\frac{1}{2}$ Substituting -2 for x

$f'(0)=\frac{1}{2}$ Substituting 0 for x

$f'(1)=\frac{1}{2}$ Substituting 1 for x

11. $f(x)=x^2+x$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{(x+h)^2+(x+h)-(x^2+x)}{h}$$
$$=\frac{x^2+2xh+h^2+x+h-x^2-x}{h}$$
$$=\frac{2xh+h^2+h}{h}$$
$$=\frac{h(2x+h+1)}{h}$$
$$=2x+h+1 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(2x+h+1)=2x+1$$

Thus, $f'(x)=2x+1$.

d) Find the values of the derivative by making the appropriate substitutions.

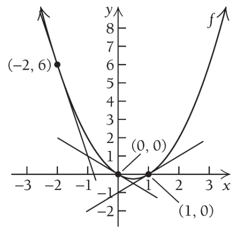
$f'(-2)=2(-2)+1=-3$

$f'(0)=2(0)+1=1$

$f'(1)=2(1)+1=3$

12. $f(x) = x^2 - x$

a), b)



c) Find the simplified difference quotient first. Referring to Exercise Set 1.3, Exercise 5 we know the simplified difference quotient is:

$$\frac{f(x+h) - f(x)}{h} = 2x + h - 1$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0}(2x + h - 1)$$
$$= 2x - 1$$

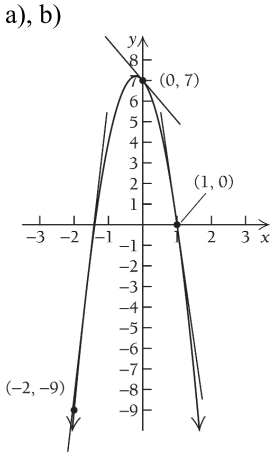
Thus, $f'(x) = 2x - 1$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2) = 2(-2) - 1 = -5$$
$$f'(0) = 2(0) - 1 = -1$$
$$f'(1) = 2(1) - 1 = 1$$

13. $f(x) = -5x^2 - 2x + 7$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{\left(-5(x+h)^2 - 2(x+h) + 7\right) - \left(-5x^2 - 2x + 7\right)}{h}$$
$$= \frac{-5\left(x^2 + 2xh + h^2\right) - 2x - 2h + 7 + 5x^2 + 2x - 7}{h}$$
$$= \frac{-5x^2 - 10xh - 5h^2 - 2x - 2h + 7 + 5x^2 + 2x - 7}{h}$$
$$= \frac{-10xh - 5h^2 - 2h}{h}$$
$$= \frac{h(-10x - 5h - 2)}{h}$$
$= -10x - 5h - 2$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0}(-10x - 5h - 2)$$
$$= -10x - 2$$

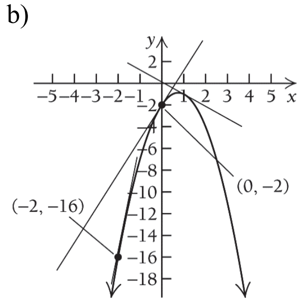
Thus, $f'(x) = -10x - 2$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2) = -10(-2) - 2 = 18$$
$$f'(0) = -10(0) - 2 = -2$$
$$f'(1) = -10(1) - 2 = -12$$

14. $f(x) = -2x^2 + 3x - 2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(-2(x+h)^2+3(x+h)-2\right)-\left(-2x^2+3x-2\right)}{h}$$
$$=\frac{-2\left(x^2+2xh+h^2\right)+3x+3h-2+2x^2-3x+2}{h}$$
$$=\frac{-2x^2-4xh-2h^2+3x+3h-2+2x^2-3x+2}{h}$$
$$=\frac{-4xh-2h^2+3h}{h}$$
$$=\frac{h(-4x-2h+3)}{h}$$
$$=-4x-2h+3 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(-4x-2h+3)$$
$$=-4x+3$$

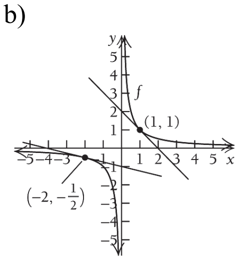
Thus, $f'(x)=-4x+3$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2)=-4(-2)+3=11$$
$$f'(0)=-4(0)+3=3$$
$$f'(1)=-4(1)+3=-1$$

15. $f(x)=\frac{1}{x}$

a), b)



There is no tangent line for $x=0$

c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(\frac{1}{x+h}\right)-\left(\frac{1}{x}\right)}{h}$$
$$=\frac{\left(\frac{1}{x+h}\cdot\frac{x}{x}\right)-\left(\frac{1}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h}$$
$$=\frac{\left(\frac{x}{x(x+h)}\right)-\left(\frac{(x+h)}{x(x+h)}\right)}{h}$$
$$=\frac{\frac{-h}{x(x+h)}}{h}$$
$$=\frac{-h}{x(x+h)}\cdot\frac{1}{h}$$
$$=\frac{-1}{x(x+h)} \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{-1}{x(x+h)}\right)$$
$$=\frac{-1}{x(x+0)}$$
$$=\frac{-1}{x^2}$$

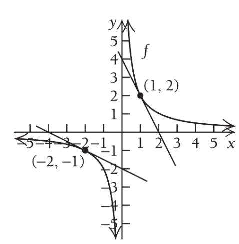
Thus, $f'(x)=\frac{-1}{x^2}$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2)=\frac{-1}{(-2)^2}=-\frac{1}{4}$$
$$f'(0)=\frac{-1}{(0)^2}; \text{Thus, } f'(0) \text{ does not exist.}$$
$$f'(1)=\frac{-1}{(1)^2}=-1$$

16. $f(x)=\frac{2}{x}$

a), b)



There is no tangent line for $x=0$.

c) Find the simplified difference quotient first. Referring to Exercise Set 1.3, Exercise 8, we know that the simplified difference quotient is:

$$\frac{f(x+h)-f(x)}{h}=\frac{-2}{x(x+h)}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{-2}{x(x+h)}\right)$$
$$=\frac{-2}{x^2}$$

Thus, $f'(x)=\frac{-2}{x^2}$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2)=\frac{-2}{(-2)^2}=-\frac{1}{2}$$

$f'(0)=\frac{-2}{(0)^2}$; Thus, $f'(0)$ does not exist.

$$f'(1)=\frac{-2}{(1)^2}=-2$$

17. From Example 2 we know that $f'(x)=3x^2$.

a) $f'(-2)=3(-2)^2=12$, so the slope of the line tangent to the curve at $(-2,-8)$ is 12. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-(-8)=12(x-(-2))$$
$$y+8=12x+24$$
$$y=12x+16$$

b) $f'(0)=3(0)^2=0$, so the slope of the line tangent to the curve at $(0,0)$ is 0. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-(0)=0(x-(0))$$
$$y=0$$

c) $f'(4)=3(4)^2=48$, so the slope of the line tangent to the curve at $(4,64)$ is 48. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-64=48(x-4)$$
$$y-64=48x-192$$
$$y=48x-128$$

18. From Example 1 we know that $f'(x)=2x$.

a) At $(3,9)$, $f'(3)=2(3)=6$.

$$y-y_1=m(x-x_1)$$
$$y-9=6(x-3)$$
$$y-9=6x-18$$
$$y=6x-9$$

b) At $(-1,1)$, $f'(-1)=2(-1)=-2$.

$$y-y_1=m(x-x_1)$$
$$y-1=-2(x-(-1))$$
$$y-1=-2x-2$$
$$y=-2x-1$$

c) At $(10,100)$, $f'(10)=2(10)=20$.

$$y-y_1=m(x-x_1)$$
$$y-100=20(x-10)$$
$$y-100=20x-200$$
$$y=20x-100$$

19. From Exercise 16 we know that $f'(x)=\frac{-2}{x^2}$.

a) $f'(1)=\frac{-2}{(1)^2}=-2$, so the slope of the line tangent to the curve at $(1,2)$ is -2. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-2=-2(x-1)$$
$$y-2=-2x+2$$
$$y=-2x+4$$

b) $f'(-1)=\frac{-2}{(-1)^2}=-2$, so the slope of the line tangent to the curve at $(-1,-2)$ is -2.

We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-(-2)&=-2(x-(-1))\\ y+2&=-2x-2\\ y&=-2x-4 \end{aligned}$$

c) $f'(100)=\frac{-2}{(100)^2}=-0.0002$, so the slope of the line tangent to the curve at $(100,0.02)$ is -0.0002. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-0.02&=-0.0002(x-100)\\ y-0.02&=-0.0002x+0.02\\ y&=-0.0002x+0.04 \end{aligned}$$

20. First, we find $f'(x)$.

$$\begin{aligned} \frac{f(x+h)-f(x)}{h}&=\frac{\left(\frac{-1}{x+h}\right)-\left(\frac{-1}{x}\right)}{h}\\ &=\frac{\left(\frac{-1}{x+h}\cdot\frac{x}{x}\right)-\left(\frac{-1}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h}\\ &=\frac{\left(\frac{-x}{x(x+h)}\right)-\left(\frac{-x-h}{x(x+h)}\right)}{h}\\ &=\frac{\frac{h}{x(x+h)}}{h}\\ &=\frac{1}{x(x+h)} \end{aligned}$$

Taking the limit as $h\to 0$ we have:

$$\begin{aligned} \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}&=\lim_{h\to 0}\frac{1}{x(x+h)}\\ &=\frac{1}{x^2} \end{aligned}$$

Thus, $f'(x)=\frac{1}{x^2}$.

a) At $(-1,1)$: $f'(-1)=\frac{1}{(-1)^2}=1$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-1&=1(x-(-1))\\ y-1&=x+1\\ y&=x+2 \end{aligned}$$

b) At $\left(2,-\frac{1}{2}\right)$: $f'(2)=\frac{1}{(2)^2}=\frac{1}{4}$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-\left(-\frac{1}{2}\right)&=\frac{1}{4}(x-2)\\ y+\frac{1}{2}&=\frac{1}{4}x-\frac{1}{2}\\ y&=\frac{1}{4}x-1 \end{aligned}$$

c) At $\left(-5,\frac{1}{5}\right)$: $f'(-5)=\frac{1}{(-5)^2}=\frac{1}{25}$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-\left(\frac{1}{5}\right)&=\frac{1}{25}(x-(-5))\\ y-\frac{1}{5}&=\frac{1}{25}x+\frac{1}{5}\\ y&=\frac{1}{25}x+\frac{2}{5} \end{aligned}$$

21. First, we will find $f'(x)$

$$\begin{aligned} &\frac{f(x+h)-f(x)}{h}\\ &=\frac{\left((x+h)^2-2(x+h)\right)-\left(x^2-2x\right)}{h}\\ &=\frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h}\\ &=\frac{2xh+h^2-2h}{h}\\ &=\frac{h(2x+h-2)}{h}\\ &=2x+h-2 \quad \text{Simplified difference quotient} \end{aligned}$$

$$\begin{aligned} f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &=\lim_{h\to 0}(2x+h-2)\\ &=2x-2 \end{aligned}$$

Thus, $f'(x)=2x-2$.

The solution is continued on the next page.

a) $f'(-2)=2(-2)-2=-6$, so the slope of the line tangent to the curve at $(-2,8)$ is -6. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-8=-6(x-(-2))$$
$$y-8=-6x-12$$
$$y=-6x-4$$

b) $f'(1)=2(1)-2=0$, so the slope of the line tangent to the curve at $(1,-1)$ is 0. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-(-1)=0(x-1)$$
$$y+1=0$$
$$y=-1$$

c) $f'(4)=2(4)-2=6$, so the slope of the line tangent to the curve at $(4,8)$ is 6. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-8=6(x-4)$$
$$y-8=6x-24$$
$$y=6x-16$$

22. First, we find $f'(x)$:

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(4-(x+h)^2\right)-\left(4-x^2\right)}{h}$$
$$=\frac{4-\left(x^2+2xh+h^2\right)-4+x^2}{h}$$
$$=\frac{4-x^2-2xh-h^2-4+x^2}{h}$$
$$=\frac{-2xh-h^2}{h}$$
$$=\frac{h(-2x-h)}{h}$$
$$=-2x-h \quad \text{Simplified difference quotient}$$

Thus,

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to0}(-2x-h)$$
$$=-2x$$

a) At $(-1,3)$, $f'(-1)=-2(-1)=2$.

$$y-y_1=m(x-x_1)$$
$$y-3=2(x-(-1))$$
$$y-3=2x+2$$
$$y=2x+5$$

b) At $(0,4)$, $f'(0)=-2(0)=0$.

$$y-y_1=m(x-x_1)$$
$$y-4=0(x-0)$$
$$y-4=0$$
$$y=4$$

c) At $(5,-21)$, $f'(5)=-2(5)=-10$.

$$y-y_1=m(x-x_1)$$
$$y-(-21)=-10(x-5)$$
$$y+21=-10x+50$$
$$y=-10x+29$$

23. Find the simplified difference quotient for $f(x)=mx+b$ first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{m(x+h)+b-(mx+b)}{h}$$
$$=\frac{mx+mh+b-mx-b}{h}$$
$$=\frac{mh}{h}$$
$$=m \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to0$ using the simplified difference quotient.

$$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}(m)$$
$$=m$$

Thus, $f'(x)=m$.

24. $f(x) = ax^2 + bx$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{\left(a(x+h)^2 + b(x+h)\right) - \left(ax^2 + bx\right)}{h}$$
$$= \frac{a\left(x^2 + 2hx + h^2\right) + bx + bh - ax^2 - bx}{h}$$
$$= \frac{ax^2 + 2ahx + ah^2 + bx + bh - ax^2 - bx}{h}$$
$$= \frac{2ahx + ah^2 + bh}{h}$$
$$= \frac{h(2ax + ah + b)}{h}$$
$$= 2ax + ah + b$$

Next we take the limit as $h \to 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} (2ax + ah + b)$$
$$= 2ax + b$$

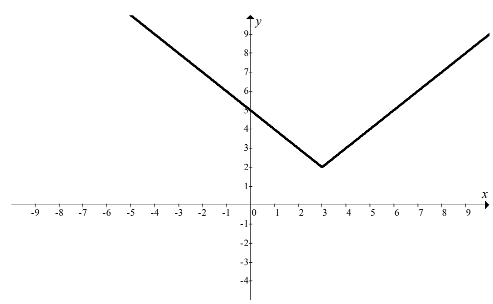
25. If a function has a "corner," it will not be differentiable at that point. Thus, the function is not differentiable at x_3, x_4, x_6. The function has a vertical tangent at x_{12}. Vertical lines have undefined slope, hence the function is not differentiable at x_{12}. Also, if a function is discontinuous at some point *a*, then it is not differentiable at *a*. The function is discontinuous at the point x_0, thus it is not differentiable at x_0.
Therefore, the graph is not differentiable at the points $x_0, x_3, x_4, x_6, x_{12}$.

26. The function is not differentiable at x_2, x_4, x_5 because there is a "corner" at each of those points. The function is not differentiable at x_7, x_8 because the function is discontinuous at those points.

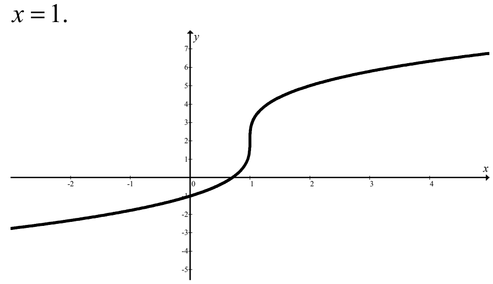
27. If a function has a "corner," it will not be differentiable at that point. Thus, the function is not differentiable at x_3. The function has a vertical tangent at x_1. Vertical lines have undefined slope, hence the function is not differentiable at x_1. Also, if a function is discontinuous at some point *a*, then it is not differentiable at *a*. The function is discontinuous at the points x_1, x_2, x_4, thus it is not differentiable at x_1, x_2, x_4.
Therefore, the graph is not differentiable at the points x_1, x_2, x_3, x_4.

28. The function is not differentiable at $x_1, x_2, x_3, x_4, x_6, x_9, x_{10}$ because there is a "corner" at each of those points. The function is not differentiable at x_5, x_7 because the function is discontinuous at those points.

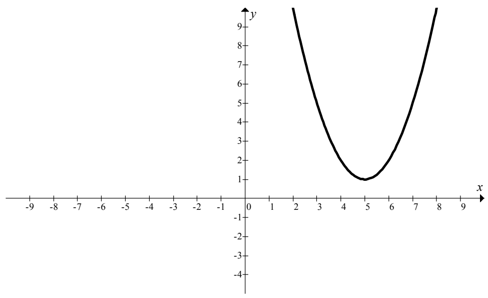
29. The following graph is continuous but not differentiable, at $x = 3$.



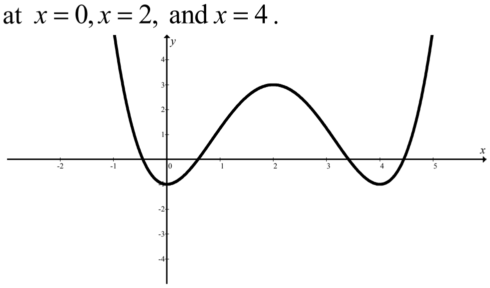
30. The following graph is continuous with no corners for all *x* but is not differentiable at $x = 1$.



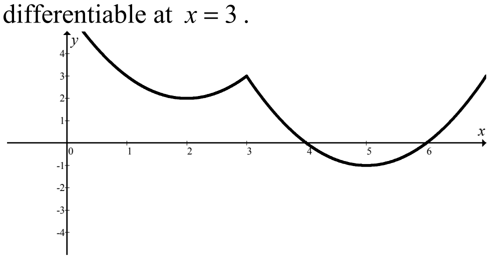
31. The following graph has a horizontal tangent line at $x = 5$.



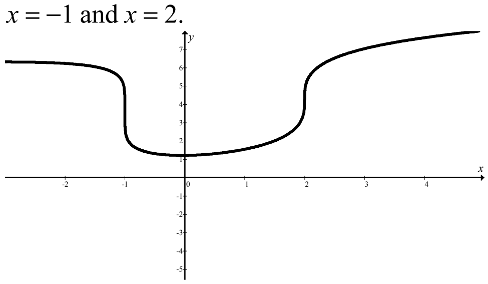
32. The following graph has horizontal tangent line at $x = 0, x = 2,$ and $x = 4$.



33. The following graph has horizontal tangent lines at $x = 2$ and $x = 5$ and is continuous but not differentiable at $x = 3$.



34. The following graph is continuous with no corners for all x but not differentiable at $x = -1$ and $x = 2$.



35. The postage function does not have any "corners" nor does it have any vertical tangents. However it is discontinuous at all natural numbers. Therefore the postage function is not differentiable for 1, 2, 3, 4, … 12.

36. The taxi cab function is not differentiable at 0, 0.2, 0.4, 0.6, and so on.

37. The largest increase occurs on December 13th. On this day rate of increase was 16 points per day. The greatest decrease occurred on December 10th. On this day rate of decrease was 53 points per day.

38. The graph is not differentiable at each of the points $x = 9, 10, 11, 12, 13$ because there is a "corner" at each of the values.

39. The lines L_2, L_3, L_4, L_6 appear to be tangent lines. The slopes appear to be the same as the instantaneous rate of change of the function at the indicated points.

40. The graph is left to the student. As the points Q approach P, the slopes of the secant lines approach the slope of the tangent line at P.

41. $f(x) = x^4$

We found the simplified difference quotient in Exercise 49 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$. Thus,

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} 4x^3 + 6x^2h + 4xh^2 + h^3 \\ &= 4x^3 \end{aligned}$$

42. $f(x) = \dfrac{1}{1-x}$

We found the simplified difference quotient in Exercise 53 of Exercise Set 1.3. We find the limit of the difference quotient as $h \to 0$. Taking the limit, we have:

$$\begin{aligned} \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{1}{(1-x-h)(1-x)} \\ &= \frac{1}{(1-x-0)(1-x)} \\ &= \frac{1}{(1-x)^2} \end{aligned}$$

Thus, $f'(x) = \dfrac{1}{(1-x)^2}$.

43. $f(x)=x^5$

We found the simplified difference quotient in Exercise 50 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$. Thus,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0} 5x^4+10x^3h+10x^2h^2+5xh^3+h^4$$
$$=5x^4$$

44. $f(x)=\frac{1}{x^2}$

We found the simplified difference quotient in Exercise 52 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{-2x-h}{x^2(x+h)^2}$$
$$=\frac{-2x}{x^2(x+0)^2}$$
$$=\frac{-2x}{x^4}$$
$$=\frac{-2}{x^3}$$

Thus, $f'(x)=\frac{-2}{x^3}$.

45. $f(x)=\sqrt{x}$

We found the simplified difference quotient in Example 8 of Section 1.3. We now find the limit of the difference quotient as $h \to 0$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$$
$$=\frac{1}{\sqrt{x+0}+\sqrt{x}}$$
$$=\frac{1}{2\sqrt{x}}$$

Thus, $f'(x)=\frac{1}{2\sqrt{x}}$

46. $f(x)=\sqrt{2x+1}$

We found the simplified difference quotient in Exercise 55 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$ at the top of the next column.

Finding the limit of the difference quotient, we have:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{2}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$$
$$=\frac{2}{\sqrt{2(x+0)+1}+\sqrt{2x+1}}$$
$$=\frac{2}{\sqrt{2x+1}+\sqrt{2x+1}}$$
$$=\frac{2}{2\sqrt{2x+1}}$$
$$=\frac{1}{\sqrt{2x+1}}$$

Thus, $f'(x)=\frac{1}{\sqrt{2x+1}}$.

47. $f(x)=\frac{1}{\sqrt{x}}$

We found the simplified difference quotient in Exercise 56 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$
$$=\frac{-1}{\sqrt{x}\sqrt{x+0}\left(\sqrt{x}+\sqrt{x+0}\right)}$$
$$=\frac{-1}{x\left(2\sqrt{x}\right)}$$
$$=\frac{-1}{2x\sqrt{x}}$$

Thus, $f'(x)=\frac{-1}{2x\sqrt{x}}$.

48. $f(x)=ax^2+bx+c$

We found the simplified difference quotient in Exercise 47 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$. Thus,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0} 2ax+ah+b$$
$$=2ax+b$$

49. a) The domain of the rational function is restricted to those input values that do not result in division by 0. The domain for $f(x)=\dfrac{x^2-9}{x+3}$ consists of all real numbers except -3. Since $f(-3)$ does not exist, the function is not continuous at -3. Thus, the function is not differentiable at $x=-3$.

b) Answers will vary. The simplest way of finding $f'(4)$ is to use the nDeriv function on your calculator. However, without using advanced technology the easiest way is to approximate the difference quotient using a very small value of h, and allowing the calculator to perform the basic computations. We illustrate using $h=0.0001$. The difference quotient will be

$$\frac{f(x+h)-f(x)}{h}=\frac{f(4+0.0001)-f(4)}{0.0001}$$

Using the calculator to evaluate the function we have:

$f(4.0001)=1.0001$

$f(4)=1$

Plugging in these values we have:

$$\frac{f(4+0.0001)-f(4)}{0.0001}=\frac{1.0001-1}{0.0001}=1$$

Therefore, $f'(4)=1$.

50. a) The domain of the rational function is restricted to those input values that do not result in division by 0. The domain for $g(x)=\dfrac{x^2+x}{2x}$ consists of all real numbers except 0. Since $g(0)$ does not exist, the function is not continuous at 0. Thus, the function is not differentiable at $x=0$.

b) Answers will vary. The simplest way of finding $g'(3)$ is to use the nDeriv function on your calculator. However, without using advanced technology the easiest way is to approximate the difference quotient using a very small value of h, and allowing the calculator to perform the basic computations. We illustrate using $h=0.0001$. The difference quotient will be

$$\frac{g(x+h)-g(x)}{h}=\frac{g(3+0.0001)-g(3)}{0.0001}$$

Using the calculator to evaluate the function we have:

$g(3.0001)=18.00165005$

$g(4)=18$

Plugging in these values we have:

$$\frac{g(3+0.0001)-g(3)}{0.0001}\approx\frac{18.00165-18}{0.0001}\approx 16.5$$

Therefore, $g'(3)=16.5$.

51. a) Looking at the graph of the function, we see there is a "corner" when $x=3$. Therefore, $k(x)=|x-3|+2$ is not differentiable at $x=3$.

b) Using the piecewise definition of

$$k(x)=|x-3|+2=\begin{cases}-(x-3)+2, & \text{for } x<3\\ (x-3)+2, & \text{for } x\ge 3\end{cases}$$

We notice that:

$$k'(x)=\begin{cases}-1, & \text{for } x<3\\ 1, & \text{for } x>3\end{cases}$$

Therefore,

$k'(0)=-1;\ k'(1)=-1;$

$k'(4)=1;\quad k'(10)=1.$

The "shortcut" is noticing that this function is a linear function with slope $m=-1$ for $x<3$ and slope $m=1$ for $x>3$.

52. a) Looking at the graph of the function, we see there is a "corner" when $x=-5$. Therefore, $k(x)=2|x+5|$ is not differentiable at $x=-5$.

b) Using the piecewise definition of

$$k(x)=2|x+5|=\begin{cases}-2(x+5), & \text{for } x<-5\\ 2(x+5), & \text{for } x\ge -5\end{cases}$$

We notice that:

$$k'(x)=\begin{cases}-2, & \text{for } x<-5\\ 2, & \text{for } x>-5\end{cases}$$

Therefore,

$k'(-10)=-2;\quad k'(-7)=-2;$

$k'(-2)=2;\qquad k'(0)=2.$

The "shortcut" is noticing that this function is a linear function with slope $m=-2$ for $x<-5$ and slope $m=2$ for $x>5$.

53. The error was made when the student did not determine the implied domain of the function.

$$f(x)=\frac{x^2+4x+3}{x+1}$$

is undefined at $x=-1$. Therefore, $f(x)$ is not differentiable at $x=-1$. Once the domain is properly defined, the student can find the derivative of the function.

54. The function $g(x)=\sqrt[3]{x}$ has a vertical slope at $x=0$. Therefore, $g'(x)$ is not defined at $x=0$. The correct conclusion is that $g(x)$ is differentiable for all real numbers x except $x=0$.

55. a) The function $F(x)$ is continuous at $x=2$, because

1) $F(2)$ exists, $F(2)=5$
2) $\lim_{x\to2^-}F(x)=5$ and $\lim_{x\to2^+}F(x)=5$,
Therefore,
$\lim_{x\to2}F(x)=5$
3) $\lim_{x\to2}F(x)=5=F(2)$.

b) The function $F(x)$ is not differentiable at $x=2$ because there is a "corner" at $x=2$.

56. a) The function $G(x)$ is continuous at $x=1$, because

1) $G(1)$ exists, $G(1)=1$
2) $\lim_{x\to1}G(x)=1$
3) $\lim_{x\to1}G(x)=1=G(1)$.

b) The function $G(x)$ is differentiable at $x=1$. $G'(1)=3$.

57. In order for $H(x)$ to be differentiable at $x=3$. $H(x)$ must be continuous at $x=3$. It must also be "smooth" at $x=3$ which means the slope as x approaches 3 from the left must equal the slope as x approaches 3 from the right.

For $x\le3$

We find the derivative by differentiating the piece of the function that is defined on the interval $x\le3$. Therefore,

$$H'(x)=\frac{d}{dx}\left(2x^2-x\right)$$
$$=4x-1.$$

Therefore, when $x=3$

$H'(3)=4(3)-1=11$.

For $x>3$ the derivative is given by

$$H'(x)=\frac{d}{dx}(mx+b)$$
$$=m.$$

Using this information, we know in order for the slope as x approaches from the right to equal the slope as x approaches from the left, we must have: $m=11$.

We also know:

$\lim_{x\to3^-}H(x)=15$, since $H(3)=15$. In order for $H(x)$ to be continuous, we must have $\lim_{x\to3^+}H(x)=15$.

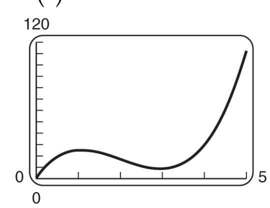
Using the above information and substituting $m=11$ we have:

$$\lim_{x\to3^+}11(x)+b=15$$
$$11(3)+b=15$$
$$33+b=15$$
$$b=-18$$

Therefore, the values $m=11$ and $b=-18$ will make $H(x)$ differentiable at $x=3$.

58-63. Left to the student.

64. a) $V(t)=5t^3-30t^2+45t+5\sqrt{t}$



b) First we find the function values

$$V(1)=5(1)^3-30(1)^2+45(1)+5\sqrt{1}$$
$$=25$$
$$V(5)=5(5)^3-30(5)^2+45(5)+5\sqrt{5}$$
$$\approx111.180$$

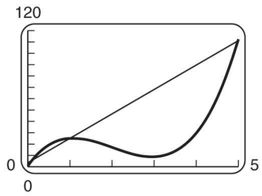
We can find the slope of the secant line passing through the two points.

$$m=\frac{V(5)-V(1)}{5-1}$$
$$=\frac{111.180-25}{5-1}$$
$$=\frac{86.180}{4}$$
$$=21.545$$

The solution is continued on the next page.

Using the information from the previous page and the point-slope equation we have:

$$V - V_1 = m(t - t_1)$$
$$V - 25 = 21.545(t - 1)$$
$$V - 25 = 21.545t - 21.545$$
$$V = 21.545t + 3.455$$



c) The average rate of change of the investment between year 1 and year 5 is the slope of the secant line between the two points. That is, the average rate of change is \$21.545 million per year.

d) For $(1, V(1))$ and $(4, V(4))$:

From part (b) we know $V(1) = 25$.

$$V(4) = 5(4)^3 - 30(4)^2 + 45(4) + 5\sqrt{4}$$
$$= 30$$
$$m = \frac{V(4) - V(1)}{4 - 1}$$
$$= \frac{30 - 25}{4 - 1}$$
$$= \frac{5}{3} \approx 1.666667$$
$$V - V_1 = m(t - t_1)$$
$$V - 25 = 1.666667(t - 1)$$
$$V - 25 = 1.666667t - 1.666667$$
$$V = 1.666667t + 23.33333$$
$$V = 1.67t + 23.3$$

The average rate of change of the investment between year 1 and year 4 is the slope of the secant line between the two points. That is, the average rate of change is \$1.67 million per year.

For $(1, V(1))$ and $(3, V(3))$:

From part (b) we know $V(1) = 25$.

$$V(3) = 5(3)^3 - 30(3)^2 + 45(3) + 5\sqrt{3}$$
$$\approx 8.660254$$
$$m = \frac{V(3) - V(1)}{3 - 1}$$
$$= \frac{8.660254 - 25}{3 - 1}$$
$$\approx -8.169873$$
$$V - V_1 = m(t - t_1)$$
$$V - 25 = -8.169873(t - 1)$$
$$V - 25 = -8.169873t + 8.169873$$
$$V = -8.169873t + 33.169873$$
$$V = -8.17t + 33.17$$

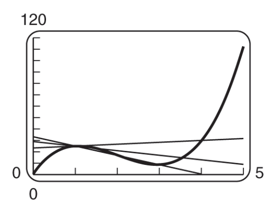
The average rate of change of the investment between year 1 and year 3 is the slope of the secant line between the two points. That is, the average rate of change is $-$\$8.17 million per year.

For $(1, V(1.5))$ and $(1.5, V(1.5))$:

From part (b) we know $V(1) = 25$.

$$V(1.5) = 5(1.5)^3 - 30(1.5)^2 + 45(1.5) + 5\sqrt{1.5}$$
$$\approx 22.998724$$
$$m = \frac{V(1.5) - V(1)}{1.5 - 1}$$
$$= \frac{22.998724 - 25}{1.5 - 1}$$
$$\approx -4.002551$$
$$V - V_1 = m(t - t_1)$$
$$V - 25 = -4.002551(t - 1)$$
$$V - 25 = -4.002551t + 4.002551$$
$$V = -4.002551t + 29.002551$$
$$V = -4.003t + 29.003$$

The average rate of change of the investment between year 1 and year 1.5 is the slope of the secant line between the two points. That is, the average rate of change is $-$\$4 million per year. The graph of the secant lines are shown:



e) Looking at the graph, the slope of the line tangent to the graph at the point $(1, V(1))$ appears to be 0.

f) The value of the investment is changing at approximately \$0 per year after 1 year.

65. There is a vertical tangent at $x = 5$, therefore, $f'(x)$ does not exist at $x = 5$.

Exercise Set 1.5

1. $y = x^8$

$$\frac{dy}{dx} = \frac{d}{dx}x^8$$
$$= 8x^{8-1} \quad \text{Theorem 1}$$
$$= 8x^7$$

2. $y = x^7$

$$\frac{dy}{dx} = 7x^{7-1} = 7x^6$$

3. $y = -0.5x$

$$\frac{dy}{dx} = \frac{d}{dx}(-0.5x)$$
$$= -0.5\frac{d}{dx}x \quad \text{Theorem 3}$$
$$= -0.5\left(1x^{1-1}\right) \quad \text{Theorem 1}$$
$$= -0.5\left(x^0\right)$$
$$= -0.5 \quad \left[a^0 = 1\right]$$

4. $y = -3x$

$$\frac{dy}{dx} = -3x^{1-1} = -3$$

5. $y = 7$ Constant function

$$\frac{dy}{dx} = \frac{d}{dx}7$$
$$= 0 \quad \text{Theorem 2}$$

6. $y = 12$

$$\frac{dy}{dx} = 0$$

7. $y = 3x^{10}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(3x^{10}\right)$$
$$= 3\frac{d}{dx}\left(x^{10}\right) \quad \text{Theorem 3}$$
$$= 3\left(10x^{10-1}\right) \quad \text{Theorem 1}$$
$$= 30x^9$$

8. $y = 2x^{15}$

$$\frac{dy}{dx} = 2\left(15x^{15-1}\right) = 30x^{14}$$

9. $y = x^{-8}$

$$\frac{dy}{dx} = \frac{d}{dx}x^{-8}$$
$$= -8x^{-8-1} \quad \text{Theorem 1}$$
$$= -8x^{-9}$$

10. $y = x^{-6}$

$$\frac{dy}{dx} = -6x^{-6-1} = -6x^{-7}$$

11. $y = 3x^{-5}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(3x^{-5}\right)$$
$$= 3\frac{d}{dx}\left(x^{-5}\right) \quad \text{Theorem 3}$$
$$= 3\left(-5x^{-5-1}\right) \quad \text{Theorem 1}$$
$$= -15x^{-6}$$

12. $y = 4x^{-2}$

$$\frac{dy}{dx} = 4\left(-2x^{-2-1}\right) = -8x^{-3}$$

13. $y = x^4 - 7x$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^4 - 7x\right)$$
$$= \frac{d}{dx}x^4 - \frac{d}{dx}7x \quad \text{Theorem 4}$$
$$= \frac{d}{dx}x^4 - 7\frac{d}{dx}x \quad \text{Theorem 3}$$
$$= 4x^{4-1} - 7\left(1x^{1-1}\right) \quad \text{Theorem 1}$$
$$= 4x^3 - 7x^0$$
$$= 4x^3 - 7 \quad \left[a^0 = 1\right]$$

14. $y = x^3 + 3x^2$

$$\frac{dy}{dx} = 3x^{3-1} + 3 \cdot 2x^{2-1} = 3x^2 + 6x$$

15. $y = 4\sqrt{x} = 4x^{1/2}$

$\frac{dy}{dx} = \frac{d}{dx}4x^{1/2}$

$= 4\frac{d}{dx}x^{1/2}$ Theorem 3

$\frac{dy}{dx} = 4\left(\frac{1}{2}x^{1/2-1}\right)$ Theorem 1

$= 2x^{-1/2}$

$= \frac{2}{x^{1/2}} = \frac{2}{\sqrt{x}}$ Properties of exponents

16. $y = 8\sqrt{x} = 8x^{1/2}$

$\frac{dy}{dx} = 8\left(\frac{1}{2}x^{1/2-1}\right) = 4x^{-1/2} = \frac{4}{x^{1/2}} = \frac{4}{\sqrt{x}}$

17. $y = x^{0.7}$

$\frac{dy}{dx} = \frac{d}{dx}x^{0.7}$

$= 0.7x^{0.7-1}$ Theorem 1

$= 0.7x^{-0.3}$

18. $y = x^{0.9}$

$\frac{dy}{dx} = 0.9x^{0.9-1} = 0.9x^{-0.1}$

19. $y = -4.8x^{1/3}$

$\frac{dy}{dx} = \frac{d}{dx}\left(-4.8x^{1/3}\right)$

$= -4.8\frac{d}{dx}\left(x^{1/3}\right)$ Theorem 3

$= -4.8\left(\frac{1}{3}x^{1/3-1}\right)$ Theorem 1

$= -1.6x^{-2/3}$

20. $y = \frac{1}{2}x^{4/5}$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{4}{5}x^{4/5-1}\right) = \frac{2}{5}x^{-1/5}$

21. $y = \frac{6}{x^4} = 6x^{-4}$

$\frac{dy}{dx} = \frac{d}{dx}\left(6x^{-4}\right)$

$= 6\frac{d}{dx}\left(x^{-4}\right)$ Theorem 3

$= 6\left(-4x^{-4-1}\right)$ Theorem 1

$= -24x^{-5}$

$= -\frac{24}{x^5}$ Properties of exponents

22. $y = \frac{7}{x^3} = 7x^{-3}$

$\frac{dy}{dx} = 7\left(-3x^{-3-1}\right) = -21x^{-4} = -\frac{21}{x^4}$

23. $y = \frac{3x}{4} = \frac{3}{4}x$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3}{4}x\right)$

$= \frac{3}{4}\cdot\frac{d}{dx}(x)$ Theorem 3

$= \frac{3}{4}\cdot\left(1x^{1-1}\right)$ Theorem 1

$= \frac{3}{4}$ Properties of exponents

24. $y = \frac{4x}{5} = \frac{4}{5}x$

$\frac{dy}{dx} = \frac{4}{5}\cdot 1x^{1-1} = \frac{4}{5}$

25. $\frac{d}{dx}\left(\sqrt[4]{x} - \frac{3}{x}\right)$

$= \frac{d}{dx}\sqrt[4]{x} - \frac{d}{dx}\frac{3}{x}$ Theorem 4

$= \frac{d}{dx}x^{1/4} - \frac{d}{dx}3x^{-1}$ Properties of exponents

$= \frac{d}{dx}x^{1/4} - 3\frac{d}{dx}x^{-1}$ Theorem 3

$= \frac{1}{4}x^{1/4-1} - 3\left(-1x^{-1-1}\right)$ Theorem 1

$= \frac{1}{4}x^{-3/4} + 3x^{-2}$

$= \frac{1}{4x^{3/4}} + \frac{3}{x^2}$

$= \frac{1}{4\sqrt[4]{x^3}} + \frac{3}{x^2}$

26. $\frac{d}{dx}\left(\sqrt[3]{x}+\frac{4}{\sqrt{x}}\right)$

$=\frac{d}{dx}\sqrt[3]{x}+\frac{d}{dx}\frac{4}{\sqrt{x}}$

$=\frac{d}{dx}x^{1/3}+\frac{d}{dx}4x^{-1/2}$

$=\frac{1}{3}x^{-2/3}-2x^{-3/2}$

$=\frac{1}{3x^{2/3}}-\frac{2}{x^{3/2}}$

$=\frac{1}{3\sqrt[3]{x^2}}-\frac{2}{\sqrt{x^3}}=\frac{1}{3\sqrt[3]{x^2}}-\frac{2}{x\sqrt{x}}$

27. $\frac{d}{dx}\left(-2\sqrt[3]{x^5}\right)$

$=-2\frac{d}{dx}\left(\sqrt[3]{x^5}\right)$ Theorem 3

$=-2\frac{d}{dx}\left(x^{5/3}\right)$

$=-2\left(\frac{5}{3}x^{5/3-1}\right)$ Theorem 1

$=-\frac{10}{3}x^{2/3}=-\frac{10\sqrt[3]{x^2}}{3}$

28. $\frac{d}{dx}\left(-\sqrt[4]{x^3}\right)$

$=-\frac{d}{dx}\left(x^{3/4}\right)=-\frac{3}{4}x^{-1/4}=-\frac{3}{4x^{1/4}}=-\frac{3}{4\sqrt[4]{x}}$

29. $\frac{d}{dx}\left(5x^2-7x+3\right)$

$=\frac{d}{dx}5x^2-\frac{d}{dx}7x+\frac{d}{dx}3$ Theorem 4

$=5\frac{d}{dx}x^2-7\frac{d}{dx}x+\frac{d}{dx}3$ Theorem 3

$=5\left(2x^{2-1}\right)-7\left(1x^{1-1}\right)+0$ Theorems 1 and 2

$=10x-7$

30. $\frac{d}{dx}\left(6x^2-5x+9\right)$

$=\frac{d}{dx}6x^2-\frac{d}{dx}5x+\frac{d}{dx}9=12x-5$

31. $f(x)=0.3x^{1.2}$

$f'(x)=\frac{d}{dx}0.3x^{1.2}$

$=0.3\frac{d}{dx}x^{1.2}$ Theorem 3

$=0.3\left(1.2x^{1.2-1}\right)$ Theorem 1

$=0.36x^{0.2}$

32. $f(x)=0.6x^{1.5}$

$f'(x)=0.6\left(1.5x^{1.5-1}\right)=0.9x^{0.5}$

33. $f(x)=\frac{3x}{4}=\frac{3}{4}x$

$f'(x)=\frac{d}{dx}\left(\frac{3}{4}x\right)$

$=\frac{3}{4}\frac{d}{dx}(x)$

$=\frac{3}{4}\left(1x^{1-1}\right)$

$=\frac{3}{4}$

34. $f(x)=\frac{2x}{3}=\frac{2}{3}x$

$f'(x)=\frac{2}{3}\left(1x^{1-1}\right)=\frac{2}{3}$

35. $f(x)=\frac{2}{5x^6}=\frac{2}{5}x^{-6}$

$f'(x)=\frac{d}{dx}\left(\frac{2}{5}x^{-6}\right)$

$=\frac{2}{5}\frac{d}{dx}\left(x^{-6}\right)$

$=\frac{2}{5}\left(-6x^{-6-1}\right)$

$=\frac{-12}{5}x^{-7}$

$=-\frac{12}{5x^7}$

36. $f(x)=\frac{4}{7x^3}=\frac{4x^{-3}}{7}=\frac{4}{7}x^{-3}$

$f'(x)=\frac{4}{7}\left(-3x^{-3-1}\right)=\frac{-12}{7}x^{-4}=-\frac{12}{7x^4}$

37. $f(x)=\frac{4}{x}-x^{3/5}=4x^{-1}-x^{3/5}$

$$f'(x)=\frac{d}{dx}\left(4x^{-1}-x^{3/5}\right)$$
$$=\frac{d}{dx}\left(4x^{-1}\right)-\frac{d}{dx}\left(x^{3/5}\right)$$
$$=4\frac{d}{dx}\left(x^{-1}\right)-\frac{d}{dx}\left(x^{3/5}\right)$$
$$=4\left(-1x^{-1-1}\right)-\left(\frac{3}{5}x^{3/5-1}\right)$$
$$=-4x^{-2}-\frac{3}{5}x^{-2/5}$$
$$=-\frac{4}{x^2}-\frac{3}{5}x^{-2/5}$$

38. $f(x)=\frac{5}{x}-x^{2/3}=5x^{-1}-x^{2/3}$

$$f'(x)=\frac{d}{dx}\left(5x^{-1}-x^{2/3}\right)$$
$$==5\left(-1x^{-1-1}\right)-\frac{2}{3}x^{2/3-1}$$
$$=-5x^{-2}-\frac{2}{3}x^{-1/3}$$
$$=-\frac{4}{x^2}-\frac{2}{3}x^{-1/3}$$

39. $f(x)=7x-14$

$$f'(x)=\frac{d}{dx}(7x-14)$$
$$=\frac{d}{dx}(7x)-\frac{d}{dx}(14)$$
$$f'(x)=7\frac{d}{dx}(x)-\frac{d}{dx}(14)$$
$$=7\left(1x^{1-1}\right)-0$$
$$=7$$

40. $f(x)=4x-7$

$$f'(x)=4\left(1x^{1-1}\right)-0=4$$

41. $f(x)=\frac{x^{3/2}}{3}=\frac{1}{3}x^{3/2}$

$$f'(x)=\frac{d}{dx}\left(\frac{1}{3}x^{3/2}\right)$$
$$=\frac{1}{3}\frac{d}{dx}\left(x^{3/2}\right)$$
$$=\frac{1}{3}\left(\frac{3}{2}x^{3/2-1}\right)$$
$$=\frac{1}{2}x^{1/2},\text{ or }\frac{\sqrt{x}}{2}$$

42. $f(x)=\frac{x^{4/3}}{4}=\frac{1}{4}x^{4/3}$

$$f'(x)=\frac{1}{4}\left(\frac{4}{3}x^{4/3-1}\right)=\frac{1}{3}x^{1/3},\text{ or }\frac{\sqrt[3]{x}}{3}$$

43. $f(x)=-0.01x^2+0.4x+50$

$$f'(x)=\frac{d}{dx}\left(-0.01x^2+0.4x+50\right)$$
$$=\frac{d}{dx}\left(-0.01x^2\right)+\frac{d}{dx}(0.4x)+\frac{d}{dx}(50)$$
$$=-0.01\frac{d}{dx}\left(x^2\right)+0.4\frac{d}{dx}(x)+\frac{d}{dx}(50)$$
$$=-0.01\left(2x^{2-1}\right)+0.4\left(1x^{1-1}\right)+0$$
$$=-0.02x+0.4$$

44. $f(x)=-0.01x^2-0.5x+70$

$$f'(x)=-0.01\left(2x^{2-1}\right)-0.5\left(1x^{1-1}\right)+0$$
$$=-0.02x-0.5$$

45. $y=x^{-3/4}-3x^{2/3}+x^{5/4}+\frac{2}{x^4}$

$$y=x^{-3/4}-3x^{2/3}+x^{5/4}+2x^{-4}$$
$$y'=\frac{d}{dx}\left(x^{-3/4}\right)-3\frac{d}{dx}\left(x^{2/3}\right)+\frac{d}{dx}\left(x^{5/4}\right)+2\frac{d}{dx}\left(x^{-4}\right)$$
$$=\left(\frac{-3}{4}x^{-3/4-1}\right)-3\left(\frac{2}{3}x^{2/3-1}\right)+$$
$$\left(\frac{5}{4}x^{5/4-1}\right)+2\left(-4x^{-4-1}\right)$$
$$=\frac{-3}{4}x^{-7/4}-2x^{-1/3}+\frac{5}{4}x^{1/4}-8x^{-5}$$
$$=\frac{-3}{4}x^{-7/4}-2x^{-1/3}+\frac{5}{4}x^{1/4}-\frac{8}{x^5}$$

46. $y = 3x^{-2/3} + x^{3/4} + x^{6/5} + \frac{8}{x^3}$

$y = 3x^{-2/3} + x^{3/4} + x^{6/5} + 8x^{-3}$

$$y' = \frac{d}{dx}\left(3x^{-2/3} + x^{3/4} + x^{6/5} + 8x^{-3}\right)$$
$$= 3\left(\frac{-2}{3}x^{-2/3-1}\right) + \left(\frac{3}{4}x^{3/4-1}\right) + \left(\frac{6}{5}x^{6/5-1}\right) + 8\left(-3x^{-3-1}\right)$$
$$= -2x^{-5/3} + \frac{3}{4}x^{-1/4} + \frac{6}{5}x^{1/5} - 24x^{-4}$$
$$= -2x^{-5/3} + \frac{3}{4}x^{-1/4} + \frac{6}{5}x^{1/5} - \frac{24}{x^4}$$

47. $y = \frac{x}{7} + \frac{7}{x} = \frac{1}{7}x + 7x^{-1}$

$$y' = \frac{d}{dx}\left(\frac{1}{7}x + 7x^{-1}\right)$$
$$= \frac{d}{dx}\left(\frac{1}{7}x\right) + \frac{d}{dx}\left(7x^{-1}\right)$$
$$= \frac{1}{7}\frac{d}{dx}\left(x^1\right) + 7\frac{d}{dx}\left(x^{-1}\right)$$
$$= \frac{1}{7}\left(1x^{1-1}\right) + 7\left(-1x^{-1-1}\right)$$
$$= \frac{1}{7} - 7x^{-2}$$
$$= \frac{1}{7} - \frac{7}{x^2}$$

48. $y = \frac{2}{x} - \frac{x}{2} = 2x^{-1} - \frac{1}{2}x$

$$y' = 2\left(-1x^{-1-1}\right) + \frac{1}{2}\left(1x^{1-1}\right)$$
$$= -2x^{-2} + \frac{1}{2} = -\frac{2}{x^2} + \frac{1}{2}$$

49. $f(x) = \sqrt{x} = x^{1/2}$

First, we find $f'(x)$

$$f'(x) = \frac{d}{dx}\left(x^{1/2}\right)$$
$$= \frac{1}{2}x^{1/2-1}$$
$$= \frac{1}{2}x^{-1/2}$$
$$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

Therefore,

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2\cdot 2} = \frac{1}{4}$$

50. $f(x) = x^2 + 4x - 5$

$$f'(x) = \frac{d}{dx}\left(x^2 + 4x - 5\right)$$
$$= \left(2x^{2-1}\right) + 4\left(1x^{1-1}\right) - 0$$
$$= 2x + 4$$

Therefore,

$$f'(10) = 2(10) + 4$$
$$= 24$$

51. $y = x + \frac{2}{x^3} = x + 2x^{-3}$

Find $\frac{dy}{dx}$ first.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + 2x^{-3}\right)$$
$$= \frac{d}{dx}(x) + 2\frac{d}{dx}\left(x^{-3}\right)$$
$$= 1x^{1-1} + 2\left(-3x^{-3-1}\right)$$
$$= 1 - 6x^{-4}$$
$$= 1 - \frac{6}{x^4}$$

Therefore,

$$\left.\frac{dy}{dx}\right|_{x=1} = 1 - \frac{6}{(1)^4}$$
$$= 1 - 6$$
$$= -5$$

52. $y = \frac{4}{x^2} = 4x^{-2}$

$$\frac{dy}{dx} = 4\left(-2x^{-2-1}\right) = -8x^{-3} = -\frac{8}{x^3}$$
$$\left.\frac{dy}{dx}\right|_{x=-2} = -\frac{8}{(-2)^3}$$
$$= -\frac{8}{(-8)} = 1$$

53. $y = \sqrt[3]{x} + \sqrt{x} = x^{\frac{1}{3}} + x^{\frac{1}{2}}$

Find $\frac{dy}{dx}$ first.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{1}{3}} + x^{\frac{1}{2}}\right)$$
$$= \frac{d}{dx}\left(x^{\frac{1}{3}}\right) + \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$
$$= \frac{1}{3}x^{\frac{1}{3}-1} + \frac{1}{2}x^{\frac{1}{2}-1}$$
$$= \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{2\sqrt{x}}$$

Therefore,

$$\left.\frac{dy}{dx}\right|_{x=64} = \frac{1}{3\sqrt[3]{(64)^2}} + \frac{1}{2\sqrt{64}}$$
$$= \frac{1}{48} + \frac{1}{16}$$
$$= \frac{1}{12}$$

54. $y = x^3 + 2x - 5$

$$\frac{dy}{dx} = 3x^{3-1} + 2x^{1-1} - 0$$
$$= 3x^2 + 2$$
$$\left.\frac{dy}{dx}\right|_{x=-2} = 3(-2)^2 + 2$$
$$= 3(4) + 2$$
$$= 14$$

55. $y = \frac{2}{5x^3} = \frac{2}{5}x^{-3}$

Find $\frac{dy}{dx}$ first.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2}{5}x^{-3}\right)$$
$$= \frac{2}{5}\frac{d}{dx}\left(x^{-3}\right)$$
$$= \frac{2}{5}\left(-3x^{-3-1}\right)$$
$$= -\frac{6}{5}x^{-4}$$
$$= -\frac{6}{5x^4}$$

Therefore,

$$\left.\frac{dy}{dx}\right|_{x=4} = -\frac{6}{5(4)^4} = -\frac{3}{640}$$

56. $y = \frac{1}{3x^4} = \frac{1}{3}x^{-4}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^{-4}\right)$$
$$= \frac{1}{3}\left(-4x^{-4-1}\right)$$
$$= -\frac{4}{3x^5}$$
$$\left.\frac{dy}{dx}\right|_{x=-1} = -\frac{4}{3(-1)^5}$$
$$= -\frac{4}{(-3)}$$
$$= \frac{4}{3}$$

57. We will need the derivative to find the slope of the tangent line at each of the indicated points. We find the derivative first.

$$f(x) = x^2 - \sqrt{x} = x^2 - x^{1/2}$$
$$f'(x) = \frac{d}{dx}\left(x^2 - x^{1/2}\right)$$
$$= \frac{d}{dx}\left(x^2\right) - \frac{d}{dx}\left(x^{1/2}\right)$$
$$= 2x^{2-1} - \frac{1}{2}x^{1/2-1}$$
$$= 2x - \frac{1}{2x^{1/2}}$$
$$= 2x - \frac{1}{2\sqrt{x}}$$

a) Using the derivative, we find the slope of the line tangent to the curve at point $(1,0)$ by evaluating the derivative at $x = 1$.

$f'(1) = 2(1) - \frac{1}{2\sqrt{1}} = 2 - \frac{1}{2} = \frac{3}{2}$. Therefore the slope of the tangent line is $\frac{3}{2}$. We use the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{3}{2}(x - 1)$$
$$y = \frac{3}{2}x - \frac{3}{2}$$

b) Using the derivative, we find the slope of the line tangent to the curve at point $(4,14)$ by evaluating the derivative at $x = 4$.

$f'(4) = 2(4) - \frac{1}{2\sqrt{4}} = 8 - \frac{1}{4} = \frac{31}{4}$.

Therefore the slope of the tangent line is $\frac{31}{4}$. We use the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 14 = \frac{31}{4}(x - 4)$$
$$y - 14 = \frac{31}{4}x - 31$$
$$y = \frac{31}{4}x - 17$$

c) Using the derivative, we find the slope of the line tangent to the curve at point $(9,78)$ by evaluating the derivative at $x = 9$.

$f'(9) = 2(9) - \frac{1}{2\sqrt{9}} = 18 - \frac{1}{6} = \frac{107}{6}$.

Therefore the slope of the tangent line is $\frac{107}{6}$. We use the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 78 = \frac{107}{6}(x - 9)$$
$$y - 78 = \frac{107}{6}x - \frac{321}{2}$$
$$y = \frac{107}{6}x - \frac{165}{2}$$

58. $f(x) = x^3 - 2x + 1$

$$f'(x) = \frac{d}{dx}(x^3 - 2x + 1)$$
$$= (3x^{3-1}) - 2(1x^{1-1}) + 0$$
$$= 3x^2 - 2$$

a) At $(2,5)$: $f'(2) = 3(2)^2 - 2 = 10$

The equation of the tangent line is:

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 10(x - 2)$$
$$y = 10x - 15$$

b) At $(-1,2)$: $f'(-1) = 3(-1)^2 - 2 = 1$

The equation of the tangent line is:

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 1(x - (-1))$$
$$y = x + 3$$

c) At: $(0,1)$ $f'(0) = 3(0)^2 - 2 = -2$

The equation of the tangent line is:

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -2(x - 0)$$
$$y = -2x + 1$$

59. We will need the derivative to find the slope of the tangent line at each of the indicated points. We find the derivative first.

$$g(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$$
$$g'(x) = \frac{d}{dx}\left(x^{\frac{2}{3}}\right)$$
$$= \frac{2}{3}x^{\frac{2}{3}-1}$$
$$= \frac{2}{3\sqrt[3]{x}}$$

a) Using the derivative, we find the slope of the line tangent to the curve at point $(-1,1)$ by evaluating the derivative at $x = -1$.

$g'(-1) = \frac{2}{3\sqrt[3]{-1}} = -\frac{2}{3}$. Therefore the slope of the tangent line is $-\frac{2}{3}$. We use the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -\frac{2}{3}(x + 1)$$
$$y - 1 = -\frac{2}{3}x - \frac{2}{3}$$
$$y = -\frac{2}{3}x + \frac{1}{3}$$

b) Using the derivative, we find the slope of the line tangent to the curve at point $(1,1)$ by evaluating the derivative at $x = 1$.

$g'(1) = \frac{2}{3\sqrt[3]{1}} = \frac{2}{3}$.

Therefore the slope of the tangent line is $\frac{2}{3}$. The solution is continued on the next page.

We use the information from the previous page and the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{2}{3}(x - 1)$$
$$y - 1 = \frac{2}{3}x - \frac{2}{3}$$
$$y = \frac{2}{3}x + \frac{1}{3}$$

c) Using the derivative, we find the slope of the line tangent to the curve at point $(8,4)$ by evaluating the derivative at $x = 8$.

$g'(8) = \frac{2}{3\sqrt[3]{8}} = \frac{1}{3}$. Therefore the slope of the tangent line is $\frac{1}{3}$.

We use the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{1}{3}(x - 8)$$
$$y - 4 = \frac{1}{3}x - \frac{8}{3}$$
$$y = \frac{1}{3}x + \frac{4}{3}$$

60. $f(x) = \frac{1}{x^2} = x^{-2}$

$$f'(x) = -2x^{-2-1}$$
$$= -2x^{-3}$$
$$= -\frac{2}{x^3}$$

a) At $(1,1)$: $f'(1) = -\frac{2}{(1)^3} = -2$

The equation of the tangent line is:

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -2(x - 1)$$
$$y = -2x + 3$$

b) At $\left(3, \frac{1}{9}\right)$: $f'(3) = -\frac{2}{(3)^3} = -\frac{2}{27}$

The equation of the tangent line is:

$$y - y_1 = m(x - x_1)$$
$$y - \frac{1}{9} = -\frac{2}{27}(x - 3)$$
$$y - \frac{1}{9} = -\frac{2}{27}x + \frac{2}{9}$$
$$y = -\frac{2}{27}x + \frac{1}{3}$$

c) At $\left(-2, \frac{1}{4}\right)$: $f'(-2) = -\frac{2}{(-2)^3} = \frac{1}{4}$

The equation of the tangent line is:

$$y - y_1 = m(x - x_1)$$
$$y - \frac{1}{4} = \frac{1}{4}(x - (-2))$$
$$y - \frac{1}{4} = \frac{1}{4}x + \frac{1}{2}$$
$$y = \frac{1}{4}x + \frac{3}{4}$$

61. $y = -x^2 + 4$

A horizontal tangent line has slope equal to 0, so we first find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(-x^2 + 4\right)$$
$$\frac{dy}{dx} = -\frac{d}{dx}x^2 + \frac{d}{dx}4$$
$$= -2x + 0$$
$$= -2x$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$-2x = 0$$
$$x = 0$$

So the horizontal tangent will occur when $x = 0$. Next we find the point on the graph. For $x = 0,\ y = -(0)^2 + 4 = 4$, so there is a horizontal tangent at the point $(0,4)$.

62. $y = x^2 - 3$

$\frac{dy}{dx} = 2x$

Solve:

$\frac{dy}{dx} = 0$

$2x = 0$

$x = 0$

For $x = 0,\ y = (0)^2 - 3 = -3$.

Therefore, there is a horizontal tangent at the point $(0, -3)$.

63. $y = x^3 - 2$

A horizontal tangent line has slope equal to 0, so we first find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^3 - 2\right)\\ &= \frac{d}{dx}x^3 - \frac{d}{dx}2\\ &= 3x^2 - 0\\ &= 3x^2\end{aligned}$$

Next, we set the derivative equal to zero and solve for x.

$\frac{dy}{dx} = 0$

$3x^2 = 0$

$x = 0$

So the horizontal tangent will occur when $x = 0$. Next we find the point on the graph. For $x = 0,\ y = (0)^3 - 2 = -2$, so there is a horizontal tangent at the point $(0, -2)$.

64. $y = -x^3 + 1$

$\frac{dy}{dx} = -3x^2$

Solve:

$\frac{dy}{dx} = 0$

$-3x^2 = 0$

$x = 0$

For $x = 0,\ y = -(0)^3 + 1 = 1$.

Therefore, there is a horizontal tangent at the point $(0, 1)$.

65. $y = 5x^2 - 3x + 8$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(5x^2 - 3x + 8\right)\\ &= \frac{d}{dx}5x^2 - \frac{d}{dx}3x + \frac{d}{dx}8\\ &= 5\left(2x^{2-1}\right) - 3\left(1x^{1-1}\right) + 0\\ &= 10x - 3\end{aligned}$$

Next, we set the derivative equal to zero and solve for x.

$\frac{dy}{dx} = 0$

$10x - 3 = 0$

$10x = 3$

$x = \frac{3}{10}$

So the horizontal tangent will occur when $x = \frac{3}{10}$. Next we find the point on the graph.

For $x = \frac{3}{10}$,

$$\begin{aligned}y &= 5\left(\frac{3}{10}\right)^2 - 3\left(\frac{3}{10}\right) + 8\\ &= 5\left(\frac{9}{100}\right) - \frac{9}{10} + 8\\ &= \frac{45}{100} - \frac{9}{10} + 8\\ &= \frac{45}{100} - \frac{9}{10}\cdot\frac{10}{10} + \frac{8}{1}\cdot\frac{100}{100}\\ &= \frac{45}{100} - \frac{90}{100} + \frac{800}{100}\\ &= \frac{45 - 90 + 800}{100}\\ &= \frac{755}{100} = \frac{151}{20}\end{aligned}$$

Therefore, there is a horizontal tangent at the point $\left(\frac{3}{10}, \frac{151}{20}\right)$, or $(0.3, 7.55)$.

66. $y = 3x^2 - 5x + 4$

$\frac{dy}{dx} = 6x - 5$

Solve:

$$\frac{dy}{dx} = 0$$
$$6x - 5 = 0$$
$$6x = 5$$
$$x = \frac{5}{6}$$

For $x = \frac{5}{6}$,

$$y = 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) + 4$$
$$= 3\left(\frac{25}{36}\right) - \frac{25}{6} + 4$$
$$= \frac{25}{12} - \frac{50}{12} + \frac{48}{12}$$
$$= \frac{23}{12}$$

Therefore, there is a horizontal tangent at the point $\left(\frac{5}{6}, \frac{23}{12}\right)$.

67. $y = -0.01x^2 + 0.4x + 50$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(-0.01x^2 + 0.4x + 50\right)$$
$$= -0.02x + 0.4 \qquad \text{See Exercise 46}$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$-0.02x + 0.4 = 0$$
$$-0.02x = -0.4$$
$$x = \frac{-0.4}{-0.02}$$
$$x = 20$$

So the horizontal tangent will occur when $x = 20$. Next we find the point on the graph.

For $x = 20$,

$$y = -0.01(20)^2 + 0.4(20) + 50$$
$$= -0.01(400) + 8 + 50$$
$$= -4 + 8 + 50$$
$$= 54$$

Therefore, there is a horizontal tangent at the point $(20, 54)$.

68. $y = -0.01x^2 - 0.5x + 70$

$\frac{dy}{dx} = -0.02x - 0.5$ See Exercise 44.

$$\frac{dy}{dx} = 0$$
$$-0.02x - 0.5 = 0$$
$$-0.02x = 0.5$$
$$x = \frac{0.5}{-0.02}$$
$$x = -25$$

For $x = -25$

$$y = -0.01(-25)^2 - 0.5(-25) + 70$$
$$= -0.01(625) + 12.5 + 70$$
$$= -6.25 + 12.5 + 70$$
$$= 76.25$$

Therefore, there is a horizontal tangent at the point $(-25, 76.25)$.

69. $y = -2x + 5$ Linear function

$\frac{dy}{dx} = -2$ Slope is -2

There are no values of x for which $\frac{dy}{dx} = 0$, so there are no points on the graph at which there is a horizontal tangent.

70. $y = 2x + 4$

$\frac{dy}{dx} = 2$

There are no values of x for which $\frac{dy}{dx} = 0$, so there are no points on the graph at which there is a horizontal tangent.

71. $y=-3$ Constant Function

$\frac{dy}{dx}=0$ Theorem 2

$\frac{dy}{dx}=0$ for all values of x, so the tangent line is horizontal for all points on the graph.

72. $y=4$

$\frac{dy}{dx}=0$ Theorem 2

$\frac{dy}{dx}=0$ for all values of x, so the tangent line is horizontal for all points on the graph.

73. $y=-\frac{1}{3}x^3+6x^2-11x-50$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx}=0$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(-\frac{1}{3}x^3+6x^2-11x-50\right)$$

$$=\frac{d}{dx}\left(-\frac{1}{3}x^3\right)+\frac{d}{dx}\left(6x^2\right)-\frac{d}{dx}(11x)-\frac{d}{dx}(50)$$

$$=-x^2+12x-11$$

Next, we set the derivative equal to zero and solve for x.

Solve:

$$\frac{dy}{dx}=0$$

$$-x^2+12x-11=0$$

$$x^2-12x+11=0$$

$$(x-1)(x-11)=0$$

$$x-1=0 \quad \text{or} \quad x-11=0$$

$$x=1 \quad \text{or} \quad x=11$$

There are two horizontal tangents. One at $x=1$ and one at $x=11$.

Next we find the points on the graph where the horizontal tangents occur.

For $x=1$,

$$y=-\frac{1}{3}(1)^3+6(1)^2-11(1)-50$$

$$=-\frac{1}{3}+6-11-50$$

$$=-\frac{166}{3}=-55\tfrac{1}{3}$$

For $x=11$,

$$y=-\frac{1}{3}(11)^3+6(11)^2-11(11)-50$$

$$=-\frac{1331}{3}+726-121-50$$

$$=\frac{334}{3}=111\tfrac{1}{3}$$

Therefore, there are horizontal tangents at the points $\left(1,-55\tfrac{1}{3}\right)$ and $\left(11,111\tfrac{1}{3}\right)$.

74. $y=-x^3+x^2+5x-1$

$$\frac{dy}{dx}=-3x^2+2x+5$$

$$\frac{dy}{dx}=0$$

$$-3x^2+2x+5=0$$

$$3x^2-2x-5=0 \quad \text{Multiply both sides by -1.}$$

$$(3x-5)(x+1)=0 \quad \text{Factor the left hand side.}$$

$$3x-5=0 \quad \text{or} \quad x+1=0$$

$$3x=5 \quad \text{or} \quad x=-1$$

$$x=\frac{5}{3} \quad \text{or} \quad x=-1$$

There are two horizontal tangents. One at $x=\frac{5}{3}$ and one at $x=-1$.

Next we find the points on the graph where the horizontal tangents occur.

For $x=-1$

$$y=-(-1)^3+(-1)^2+5(-1)-1$$

$$y=-(-1)+(1)-5-1$$

$$y=-4$$

For $x=\frac{5}{3}$

$$y=-\left(\frac{5}{3}\right)^3+\left(\frac{5}{3}\right)^2+5\left(\frac{5}{3}\right)-1$$

$$y=-\left(\frac{125}{27}\right)+\left(\frac{25}{9}\right)+\frac{25}{3}-1$$

$$y=-\frac{125}{27}+\frac{75}{27}+\frac{225}{27}-\frac{27}{27}$$

$$y=\frac{148}{27}=5\tfrac{13}{27}$$

Therefore, there are horizontal tangents at the points $\left(\frac{5}{3},5\tfrac{13}{27}\right)$ and $(-1,-4)$.

75. $y = x^3 - 6x + 1$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 - 6x + 1\right)$$
$$= \frac{d}{dx}\left(x^3\right) - \frac{d}{dx}(6x) + \frac{d}{dx}(1)$$
$$= 3x^2 - 6$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$3x^2 - 6 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

There are two horizontal tangents. One at $x = -\sqrt{2}$ and one at $x = \sqrt{2}$. Next we find the points on the graph where the horizontal tangents occur.

For $x = -\sqrt{2}$,

$$y = \left(-\sqrt{2}\right)^3 - 6\left(-\sqrt{2}\right) + 1$$
$$= -2\sqrt{2} + 6\sqrt{2} + 1$$
$$= 1 + 4\sqrt{2}$$

For $x = \sqrt{2}$,

$$y = \left(\sqrt{2}\right)^3 - 6\left(\sqrt{2}\right) + 1$$
$$= 2\sqrt{2} - 6\sqrt{2} + 1$$
$$= 1 - 4\sqrt{2}$$

Therefore, there are horizontal tangents at the points $\left(-\sqrt{2}, 1 + 4\sqrt{2}\right)$ and $\left(\sqrt{2}, 1 - 4\sqrt{2}\right)$.

76. $y = \frac{1}{3}x^3 - 3x + 2$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3 - 3x + 2\right)$$
$$= \frac{d}{dx}\left(\frac{1}{3}x^3\right) - \frac{d}{dx}(3x) + \frac{d}{dx}(2)$$
$$= x^2 - 3$$

Solve:

$$\frac{dy}{dx} = 0$$
$$x^2 - 3 = 0$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

For $x = -\sqrt{3}$

$$y = \frac{1}{3}\left(-\sqrt{3}\right)^3 - 3\left(-\sqrt{3}\right) + 2$$
$$= \frac{1}{3}\left(-3\sqrt{3}\right) - 3\left(-\sqrt{3}\right) + 2$$
$$= -\sqrt{3} + 3\sqrt{3} + 2$$
$$= 2 + 2\sqrt{3}$$

For $x = \sqrt{3}$

$$y = \frac{1}{3}\left(\sqrt{3}\right)^3 - 3\left(\sqrt{3}\right) + 2$$
$$= \frac{1}{3}\left(3\sqrt{3}\right) - 3\left(\sqrt{3}\right) + 2$$
$$= \sqrt{3} - 3\sqrt{3} + 2$$
$$= 2 - 2\sqrt{3}$$

Therefore, there are horizontal tangents at the points $\left(-\sqrt{3}, 2 + 2\sqrt{3}\right)$ and $\left(\sqrt{3}, 2 - 2\sqrt{3}\right)$.

77. $y = \frac{1}{3}x^3 - 3x^2 + 9x - 9$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3 - 3x^2 + 9x - 9\right)$$
$$= \frac{d}{dx}\left(\frac{1}{3}x^3\right) - \frac{d}{dx}\left(3x^2\right) + \frac{d}{dx}(9x) - \frac{d}{dx}(9)$$
$$= x^2 - 6x + 9$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$x^2 - 6x + 9 = 0$$
$$(x - 3)^2 = 0$$
$$x - 3 = 0$$
$$x = 3$$

There is one horizontal tangent at $x = 3$.

The solution is continued on the next page.

Using the information from the previous page, we find the point on the graph where the horizontal tangent occurs.
For $x=3$,

$$y=\frac{1}{3}(3)^3-3(3)^2+9(3)-9$$
$$=9-27+27-9=0$$

Therefore, there is horizontal tangent at the point $(3,0)$.

78. $y=\frac{1}{3}x^3+\frac{1}{2}x^2-2$

$\frac{dy}{dx}=x^2+x$

Solve:

$$\frac{dy}{dx}=0$$
$$x^2+x=0$$
$$x(x+1)=0$$
$$x=0 \quad \text{or} \quad x+1=0$$
$$x=0 \quad \text{or} \quad x=-1$$

For $x=0$

$$y=\frac{1}{3}(0)^3+\frac{1}{2}(0)^2-2$$
$$y=-2$$

For $x=-1$

$$y=\frac{1}{3}(-1)^3+\frac{1}{2}(-1)^2-2$$
$$=-\frac{1}{3}+\frac{1}{2}-2$$
$$=-\frac{11}{6}$$

Therefore, there are horizontal tangents at the points $(0,-2)$ and $\left(-1,-\frac{11}{6}\right)$.

79. $y=6x-x^2$

To find the tangent line that has slope equal to 1, so we need to find the values of x that make $\frac{dy}{dx}=1$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(6x-x^2\right)$$
$$=\frac{d}{dx}6x-\frac{d}{dx}x^2$$
$$=6-2x$$

Next, we set the derivative equal to 1 and solve for x.

$$\frac{dy}{dx}=1$$
$$6-2x=1$$
$$-2x=1-6$$
$$-2x=-5$$
$$x=\frac{5}{2}$$

So the tangent will occur when $x=\frac{5}{2}$.

Next we find the point on the graph.

For $x=\frac{5}{2}$,

$$y=6\left(\frac{5}{2}\right)-\left(\frac{5}{2}\right)^2$$
$$=15-\frac{25}{4}=\frac{35}{4}$$

The tangent line has slope 1 at $\left(\frac{5}{2},\frac{35}{4}\right)$ or $(2.5,8.75)$.

80. $y=20x-x^2$; $\frac{dy}{dx}=20-2x$

Solve:

$$\frac{dy}{dx}=1$$
$$20-2x=1$$
$$-2x=-19$$
$$x=\frac{19}{2}$$

For $x=\frac{19}{2}$,

$$y=20\left(\frac{19}{2}\right)-\left(\frac{19}{2}\right)^2$$
$$y=190-\left(\frac{361}{4}\right)$$
$$=\frac{760}{4}-\frac{361}{4}$$
$$=\frac{399}{4}$$

The tangent line has slope 1 at the point $\left(\frac{19}{2},\frac{399}{4}\right)$ or $(9.5,99.75)$.

81. $y=-0.01x^2+2x$

To find the tangent line that has slope equal to 1, we need to find the values of x that make $\frac{dy}{dx}=1$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(-0.01x^2+2x\right)$$
$$=\frac{d}{dx}\left(-0.01x^2\right)+\frac{d}{dx}(2x)$$
$$=-0.01\left(2x^{2-1}\right)+2\left(x^{1-1}\right)$$
$$=-0.02x+2$$

Next, we set the derivative equal to 1 and solve for x.

$$\frac{dy}{dx}=1$$
$$-0.02x+2=1$$
$$-0.02x=-1$$
$$x=\frac{-1}{-0.02}$$
$$x=50$$

So the tangent will occur when $x=50$. Next we find the point on the graph.

For $x=50$,

$$y=-0.01(50)^2+2(50)$$
$$=-0.01(2500)+100$$
$$=-25+100$$
$$=75$$

The tangent line has slope 1 at $(50,75)$.

82. $y=-0.025x^2+4x$; $\frac{dy}{dx}=-0.05x+4$

Solve:

$$\frac{dy}{dx}=1$$
$$-0.05x+4=1$$
$$-0.05x=-3$$
$$x=60$$

For $x=60$,

$$y=-0.025(60)^2+4(60)=-90+240=150$$

The tangent line has slope 1 at the point $(60,150)$.

83. $y=\frac{1}{3}x^3-x^2-4x+1$

To find the tangent line that has slope equal to 1, we need to find the values of x that make $\frac{dy}{dx}=1$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{3}x^3-x^2-4x+1\right)$$
$$=\frac{d}{dx}\left(\frac{1}{3}x^3\right)-\frac{d}{dx}x^2-\frac{d}{dx}4x+\frac{d}{dx}1$$
$$=x^2-2x-4$$

Next, we set the derivative equal to 1 and solve for x.

$$\frac{dy}{dx}=1$$
$$x^2-2x-4=1$$
$$x^2-2x-5=0$$

This is a quadratic equation, not readily factorable, so we use the quadratic formula where $a=1, b=4,$ and, $c=1$.

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-5)}}{2(1)}$$
$$=\frac{2\pm\sqrt{4+20}}{2}$$
$$=\frac{2\pm\sqrt{24}}{2}$$
$$=\frac{2\pm2\sqrt{6}}{2}$$
$$=1\pm\sqrt{6}$$

There are two tangent lines that have slope equal to 1. The first one occurs at $x=1+\sqrt{6}$ and the second one occurs at $x=1-\sqrt{6}$. We use the original equation to find the points on the graph.

For $x=1+\sqrt{6}$,

$$y=\frac{1}{3}\left(1+\sqrt{6}\right)^3-\left(1+\sqrt{6}\right)^2-4\left(1+\sqrt{6}\right)+1$$
$$=\frac{1}{3}\left(19+9\sqrt{6}\right)-\left(7+2\sqrt{6}\right)-4-4\sqrt{6}+1$$
$$=\frac{19}{3}+3\sqrt{6}-7-2\sqrt{6}-4-4\sqrt{6}+1$$
$$=-\frac{11}{3}-3\sqrt{6}$$

The solution is continued on the next page.

For $x = 1-\sqrt{6}$,

$$y = \frac{1}{3}\left(1-\sqrt{6}\right)^3 - \left(1-\sqrt{6}\right)^2 - 4\left(1-\sqrt{6}\right)+1$$
$$= \frac{1}{3}\left(19-9\sqrt{6}\right) - \left(7-2\sqrt{6}\right) - 4 + 4\sqrt{6}+1$$
$$= \frac{19}{3} - 3\sqrt{6} - 7 + 2\sqrt{6} - 4 + 4\sqrt{6} + 1$$
$$= -\frac{11}{3} + 3\sqrt{6}$$

Therefore, the tangent line has slope 1 at the points

$\left(1+\sqrt{6}, -\frac{11}{3} - 3\sqrt{6}\right)$ and $\left(1-\sqrt{6}, -\frac{11}{3} + 3\sqrt{6}\right)$.

84. $y = \frac{1}{3}x^3 + 2x^2 + 2x$

$$\frac{dy}{dx} = x^2 + 4x + 2$$

Solve:

$$\frac{dy}{dx} = 1$$
$$x^2 + 4x + 2 = 1$$
$$x^2 + 4x + 1 = 0$$

Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(1)}}{2(1)} \quad \text{Substituting}$$
$$= \frac{-4 \pm \sqrt{12}}{2}$$
$$= \frac{-4 \pm 2\sqrt{3}}{2} \qquad \left[\sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}\right]$$
$$= \frac{2\left(-2 \pm \sqrt{3}\right)}{2}$$
$$= -2 \pm \sqrt{3}$$

For $x = -2+\sqrt{3}$,

$$y = \frac{1}{3}\left(-2+\sqrt{3}\right)^3 + 2\left(-2+\sqrt{3}\right)^2 + 2\left(-2+\sqrt{3}\right)$$
$$= \frac{1}{3}\left(-26+15\sqrt{3}\right) + 2\left(7-4\sqrt{3}\right) - 4 + 2\sqrt{3}$$
$$= -\frac{26}{3} + 5\sqrt{3} + 14 - 8\sqrt{3} - 4 + 2\sqrt{3}$$
$$= \frac{4}{3} - \sqrt{3}$$

For $x = -2-\sqrt{3}$,

$$y = \frac{1}{3}\left(-2-\sqrt{3}\right)^3 + 2\left(-2-\sqrt{3}\right)^2 + 2\left(-2-\sqrt{3}\right)$$
$$= \frac{1}{3}\left(-26-15\sqrt{3}\right) + 2\left(7+4\sqrt{3}\right) - 4 - 2\sqrt{3}$$
$$= -\frac{26}{3} - 5\sqrt{3} + 14 + 8\sqrt{3} - 4 - 2\sqrt{3}$$
$$= \frac{4}{3} + \sqrt{3}$$

The tangent lines have slope 1 at the points

$\left(-2+\sqrt{3}, \frac{4}{3} - \sqrt{3}\right)$ and $\left(-2-\sqrt{3}, \frac{4}{3} + \sqrt{3}\right)$.

85. a) In order to find the rate of change of the circumference with respect to the radius, we must find the derivative of the function with respect to *r*.

$$C(r) = 6.28r$$
$$C'(r) = 6.28$$

b) $C'(4) = 6.28$

c) Answers will vary. $C'(4) = 6.28$ means that the circumference of the wound with a radius 4 cm will increase at a rate of 6.28 cm for each centimeter increase in the radius.

86. a) In order to find the rate of change of the area with respect to the radius, we must find the derivative of the function with respect to *r*.

$$A'(r) = \frac{d}{dr}\left(3.14r^2\right)$$
$$= 3.14\left(2r^{2-1}\right)$$
$$= 6.28r$$

b) $A'(3) = 6.28(3) = 18.84$.

c) Answers will vary. $A'(3) = 18.84$ means that the area of the wound with a radius 3 cm will increase at a rate of 18.84 cm^2 for each centimeter increase in the radius.

87. $w(t)=8.15+1.82t-0.0596t^2+0.000758t^3$

a) In order to find the rate of change of weight with respect to time, we take the derivative of the function with respect to *t*.

$$w'(t)$$
$$=\frac{d}{dt}\left(8.15+1.82t-0.0596t^2+0.000758t^3\right)$$
$$=0+1.82-0.0596(2t)+0.000758\left(3t^2\right)$$
$$=1.82-0.1192t+0.002274t^2$$

Therefore, the rate of change of weight with respect to time is given by:

$$w'(t)=1.82-0.1192t+0.002274t^2$$

b) The weight of the baby at age 10 months can be found by evaluating the function when $t=10$.

$$w(10)=8.15+1.82(10)-0.0596(10)^2+0.000758(10)^3$$
$$\approx 21.148 \quad \text{Using a calculator}$$

Therefore, a 10 month old boy weighs approximately 21.148 pounds.

c) The rate of change of the baby's weight with respect to time at age of 10 months can be found by evaluating the derivative when $t=10$.

$$w'(10)$$
$$=1.82-0.1192(10)+0.002274(10)^2$$
$$\approx 0.8554$$

A 10 month old boys weight will be increasing at a rate of 0.86 pounds per month.

88. $T(t)=-0.1t^2+1.2t+98.6$

a) $T'(t)=-0.2t+1.2$

b) Evaluate *T* when $t=1.5$

$$T(1.5)=-0.1(1.5)^2+1.2(1.5)+98.6$$
$$=100.175$$

The temperature of the ill person after 1.5 days is 100.2 degrees Fahrenheit.

c) Evaluate $T'(t)$ when $t=1.5$.

$$T'(1.5)=-0.2(1.5)+1.2$$
$$=0.9$$

The ill person's temperature is increasing 0.9 degrees Fahrenheit per day after 1.5 days.

89. $R(v)=\frac{6000}{v}=6000v^{-1}$

a) Using the power rule, we take the derivative of *R* with respect to *v*.

$$R'(v)=6000\left(-1v^{-1-1}\right)$$
$$=-6000v^{-2}$$
$$=-\frac{6000}{v^2}$$

The rate of change of heart rate with respect to the output per beat is

$$R'(v)=-\frac{6000}{v^2}.$$

b) To find the heart rate at $v=80$ mL per beat, we evaluate the function $R(v)$ when $v=80$.

$$R(80)=\frac{6000}{80}=75.$$

The heart rate is 75 beats per minute when the output per beat is 80 mL per beat.

c) To find the rate of change of the heart beat at $v=80$ mL per beat, we evaluate the derivative $R'(v)$ at $v=80$.

$$R'(80)=-\frac{6000}{80^2}$$
$$R'(80)=-\frac{15}{16}$$
$$=-0.9375$$

The heart rate is decreasing at a rate of 0.94 beats per minute when the output per beat is 80 mL per beat.

90. $S(r)=\frac{1}{r^4}=r^{-4}$

a) $S'(r)=\frac{d}{dr}\left(r^{-4}\right)=-4r^{-5}=-\frac{4}{r^5}$

b) $S(1.2)=\frac{1}{(1.2)^4}\approx 0.48225309$

The resistance when $r=1.2$ mm is 0.48225309.

c) $S'(r)=-\frac{4}{(0.8)^5}\approx -12.20703125$

The resistance, *S,* is changing with respect to *r* at an approximate rate of −12.2 per mm when $r=0.8$ mm.

91. a) Using the power rule, we find the growth rate $\frac{dP}{dt}$.

$$\frac{dP}{dt} = \frac{d}{dt}\left(100,000 + 2000t^2\right)$$
$$= 0 + 2000(2t)$$
$$= 4000t$$

b) Evaluate the function P when $t = 10$.

$$P(10) = 100,000 + 2000(10)^2$$
$$= 100,000 + 2000(100)$$
$$= 300,000$$

The population of the city will be 300,000 people after 10 years.

c) Evaluate the derivative $P'(t)$ when $t = 10$.

$$\left.\frac{dP}{dt}\right|_{t=10} = P'(10) = 4000(10) = 40,000$$

The population's growth rate after 10 years is 40,000 people per year.

d) ✎ Answers will vary. $P'(10) = 40,000$ means that after 10 years, the city's population is growing at a rate of 40,000 people per year.

92. $A(t) = 0.08t + 19.7$

a) $A'(t) = 0.08$

b) ✎ Answers will vary. The median age, *A,* of women marrying for the first time has been increasing at a rate 0.08 year per year since 1950.

93. $V = 1.22\sqrt{h} = 1.22h^{1/2}$

a) Using the power rule,

$$\frac{dV}{dh} = \frac{d}{dh}\left(1.22h^{1/2}\right)$$
$$= 1.22\left(\frac{1}{2}h^{1/2-1}\right)$$
$$= 0.61h^{-1/2}$$
$$= \frac{0.61}{h^{1/2}} = \frac{0.61}{\sqrt{h}}$$

b) Evaluate the function V when $h = 40,000$.

$$V = 1.22\sqrt{40,000}$$
$$= 244$$

A person would be able to see 244 miles to the horizon from a height of 40,000 feet.

c) Evaluate the derivative $\frac{dV}{dh}$ when $h = 40,000$.

$$\left.\frac{dV}{dh}\right|_{h=40,000} = \frac{0.61}{\sqrt{40,000}}$$
$$= \frac{0.61}{200}$$
$$= 0.00305$$

The rate of change at $h = 40,000$ is 0.0031 miles per foot.

d) ✎ Answers will vary. From part (a), we find that the distance that one can see to the horizon from height h increases $\frac{0.61}{\sqrt{h}}$ miles for every one foot increase in height. From part (c) we find that, at a height of 40,000 feet, the distance that a person can see to the horizon increases at a rate of 0.0031 miles per foot.

94. $p(t) = 0.696t^2 - 13.290t + 61.857$

a) In 2014, $t = 2014 - 1967 = 47$. Evaluating the function at $t = 47$, we have

$$p(47) = 0.696(47)^2 - 13.290(47) + 61.857$$
$$p(47) = 974.69$$

The average ticket price in 2014 is \$974.69.

b) $\frac{dp}{dt} = p'(t) = 1.392t - 13.290$

c) Evaluate the derivative at $t = 47$.

$$p'(47) = 1.392(47) - 13.29 = 52.13$$

In 2014, the average ticket price is increasing at a rate of \$52.13 per year.

95. $f(x) = x^2 - 4x + 1$

The derivative is positive when $f'(x) > 0$.

Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left(x^2 - 4x + 1\right)$$
$$= 2x - 4$$

Next, we solve the inequality

$$f'(x) > 0$$
$$2x - 4 > 0$$
$$2x > 4$$
$$x > 2$$

Therefore, the interval for which $f'(x)$ is positive is $(2, \infty)$.

96. $f(x)=\frac{1}{3}x^3-x^2-3x+5$

The derivative is positive when $f'(x)>0$.

Find $f'(x)$.

$$f'(x)=\frac{d}{dx}\left(\frac{1}{3}x^3-x^2-3x+5\right)$$
$$=x^2-2x-3$$

Next, we solve the inequality

$$f'(x)>0$$
$$x^2-2x-3>0$$

First we find where the quadratic is equal to zero, in order to determine the intervals that we will need to test.

$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$
$$x+1=0 \quad \text{or} \quad x-3=0$$
$$x=-1 \quad \text{or} \quad x=3$$

Now we will test a value to the left of -1, between -1 and 3 and to the right of 3 to determine where the quadratic is positive or negative. We choose the values $x=-2,\ x=0,$ and, $x=4$ to test.

When $x=-2$, the derivative $f'(x)$ is

$$f'(-2)=(-2)^2-2(-2)-3=5.$$

When $x=0$, the derivative $f'(x)$ is

$$f'(0)=(0)^2-2(0)-3=-3.$$

When $x=4$, the derivative $f'(x)$ is

$$f'(4)=(4)^2-2(4)-3=5.$$

We organize the results in the table below.

Test point x	Test $x=-2$	-1	Test $x=0$	3	Test $x=4$
$f'(x)$	$f'(-2)=5$	0	$f'(0)=-3$	0	$f'(4)=5$

From the table, we can see that $f'(x)$ is positive on the interval $(-\infty,-1)$ and the interval $(3,\infty)$.

97. $y=x^4-\frac{4}{3}x^2-4$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx}=0$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^4-\frac{4}{3}x^2-4\right)$$
$$=\frac{d}{dx}x^4-\frac{d}{dx}\left(\frac{4}{3}x^2\right)-\frac{d}{dx}4$$
$$=4x^{4-1}-\frac{4}{3}\left(2x^{2-1}\right)-0$$
$$=4x^3-\frac{8}{3}x$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx}=0$$
$$4x^3-\frac{8}{3}x=0$$
$$4x\left(x^2-\frac{2}{3}\right)=0$$
$$4x=0 \quad \text{or} \quad x^2-\frac{2}{3}=0$$
$$x=0 \quad \text{or} \quad x^2=\frac{2}{3}$$
$$x=0 \quad \text{or} \quad x=\pm\sqrt{\frac{2}{3}}$$

So the horizontal tangent will occur when $x=0,\ x=\sqrt{\frac{2}{3}},$ and $x=-\sqrt{\frac{2}{3}}$. Next we find the points on the graph.

For $x=0$,

$$y=(0)^4-\frac{4}{3}(0)^2-4=-4.$$

For $x=\sqrt{\frac{2}{3}}$

$$y=\left(\sqrt{\frac{2}{3}}\right)^4-\frac{4}{3}\left(\sqrt{\frac{2}{3}}\right)^2-4$$
$$=-\frac{40}{9}$$

The solution is continued on the next page

For $x=-\sqrt{\frac{2}{3}}$

$$y=\left(-\sqrt{\frac{2}{3}}\right)^4-\frac{4}{3}\left(-\sqrt{\frac{2}{3}}\right)^2-4$$
$$=-\frac{40}{9}$$

There are three points on the graph for which the tangent line is horizontal.

$(0,-4)$, $\left(\sqrt{\frac{2}{3}},-\frac{40}{9}\right)$, and, $\left(-\sqrt{\frac{2}{3}},-\frac{40}{9}\right)$.

98. $y=2x^6-x^4-2$

$$\frac{dy}{dx}=\frac{d}{dx}\left(2x^6-x^4-2\right)$$
$$=\frac{d}{dx}2x^6-\frac{d}{dx}x^4-\frac{d}{dx}2$$
$$=2\left(6x^{6-1}\right)-\left(4x^{4-1}\right)+0$$
$$=12x^5-4x^3$$

Solve $\frac{dy}{dx}=0$

$$12x^5-4x^3=0$$
$$4x^3\left(3x^2-1\right)=0$$

$4x^3=0$ or $3x^2-1=0$

$x=0$ or $3x^2=1$

$x=0$ or $x^2=\frac{1}{3}$

$x=0$ or $x=\pm\sqrt{\frac{1}{3}}=\pm\frac{1}{\sqrt{3}}$

For $x=0$,

$$y=2(0)^6-(0)-2$$
$$=-2$$

For $x=\frac{1}{\sqrt{3}}$,

$$y=2\left(\frac{1}{\sqrt{3}}\right)^6-\left(\frac{1}{\sqrt{3}}\right)^4-2$$
$$=2\left(\frac{1}{27}\right)-\frac{1}{9}-2$$
$$=\frac{2}{27}-\frac{3}{27}-\frac{54}{27}$$
$$=-\frac{55}{27}$$

For $x=-\frac{1}{\sqrt{3}}$,

$$y=2\left(-\frac{1}{\sqrt{3}}\right)^6-\left(-\frac{1}{\sqrt{3}}\right)^4-2$$
$$=2\left(\frac{1}{27}\right)-\frac{1}{9}-2$$
$$=\frac{2}{27}-\frac{3}{27}-\frac{54}{27}$$
$$=-\frac{55}{27}$$

Therefore, there are horizontal tangents at the points $(0,-2)$, $\left(\frac{1}{\sqrt{3}},-\frac{55}{27}\right)$, and $\left(-\frac{1}{\sqrt{3}},-\frac{55}{27}\right)$.

99. $f(x)=x^5+x^3$

Taking the derivative we have:

$$f'(x)=\frac{d}{dx}\left(x^5+x^3\right)$$
$$=\frac{d}{dx}x^5+\frac{d}{dx}x^3$$
$$=5x^4+3x^2$$

Notice that $f'(x)\geq 0$ for all values of x.

Therefore, $f(x)$ is always increasing.

100. $g(x)=x^3+2x$

Taking the derivative we have:

$$g'(x)=\frac{d}{dx}\left(-2x-x^3\right)$$
$$=-2\frac{d}{dx}x-\frac{d}{dx}x^3$$
$$=-2x^{1-1}-3x^{3-1}$$
$$=-2-3x^2$$

Notice that $g'(x)\leq 0$ for all values of x.

Therefore, $g(x)$ is always decreasing.

101. $k(x)=\frac{1}{x^2},\ x\neq 0$

Taking the derivative we have:

$$k'(x)=\frac{d}{dx}\left(\frac{1}{x^2}\right)$$
$$=\frac{d}{dx}x^{-2}$$
$$=-2x^{-2-1}$$
$$=-2x^{-3}=-\frac{2}{x^3}$$

Notice that $k'(x)<0$ for all values of x over the interval $(0,\infty)$ Therefore, $k(x)$ is always decreasing over the interval $(0,\infty)$.

102. $f(x)=x^3+ax$

Finding the derivative, we have:

$f'(x)=3x^2+a$.

When a is positive, $f'(x)>0$ for all values of x and will be always increasing. However, if a is negative, then $f'(x)<0$ for some values of x. (most noticeably $x=0$) and therefore will not be always increasing.

103. $y=(x+3)(x-2)$

First, we multiply the two binomials.

$y=(x+3)(x-2)=x^2+x-6$

Therefore,

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^2+x-6\right)$$
$$=\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}(x)-\frac{d}{dx}(6)$$
$$=2x+1$$

104. $y=(x-1)(x+1)=x^2-1$

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^2-1\right)=2x$$

105. $y=\frac{x^5-x^3}{x^2}$

First, we separate the fraction.

$$y=\frac{x^5}{x^2}-\frac{x^3}{x^2}$$
$$=x^{5-2}-x^{3-2} \qquad \left[\frac{a^m}{a^n}=a^{m-n}\right]$$
$$=x^3-x^1$$

Therefore,

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^3-x\right)$$
$$=\frac{d}{dx}x^3-\frac{d}{dx}x$$
$$=3x^2-1$$

106. $y=\frac{x^5+x}{x^2}$

First, we separate the fraction.

$$y=\frac{x^5}{x^2}+\frac{x}{x^2}$$
$$=x^{5-2}+x^{1-2}$$
$$=x^3+x^{-1}$$

Therefore,

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^3+x^{-1}\right)$$
$$=3x^2+\left(-x^{-1-1}\right)$$
$$=3x^2-x^{-2}$$
$$=3x^2-\frac{1}{x^2}$$

107. $y=\sqrt{7x}$

First, we simplify the radical.

$$y=\sqrt{7\cdot x}$$
$$=\sqrt{7}\sqrt{x} \qquad \left[\sqrt{m\cdot n}=\sqrt{m}\sqrt{n}\right]$$
$$=\sqrt{7}\left(x^{1/2}\right) \qquad \left[\sqrt[m]{a}=a^{1/m};m=2\right]$$

Therefore,

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{7}(x)^{1/2}\right)$$
$$=\sqrt{7}\frac{d}{dx}\left(x^{1/2}\right)$$
$$=\sqrt{7}\left(\frac{1}{2}x^{1/2-1}\right)$$
$$=\frac{\sqrt{7}}{2}x^{-1/2}$$
$$=\frac{\sqrt{7}}{2x^{1/2}}$$
$$=\frac{\sqrt{7}}{2\sqrt{x}}$$

108. $y=\sqrt[3]{8x}=(8x)^{1/3}=8^{1/3}\cdot x^{1/3}=2x^{1/3}$

$$\frac{dy}{dx}=\frac{2}{3}x^{-\frac{2}{3}}=\frac{2}{3x^{2/3}}=\frac{2}{3\sqrt[3]{x^2}}$$

109. $y=\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)^2$

$$y=\left(x^{1/2}-x^{-1/2}\right)^2$$
$$=\left(x^{1/2}-x^{-1/2}\right)\left(x^{1/2}-x^{-1/2}\right)$$
$$=x-2x^0+x^{-1}$$
$$=x+x^{-1}-2$$
$$\frac{dy}{dx}=\frac{d}{dx}\left(x+x^{-1}-2\right)$$
$$=1-x^{-2}$$
$$=1-\frac{1}{x^2}$$

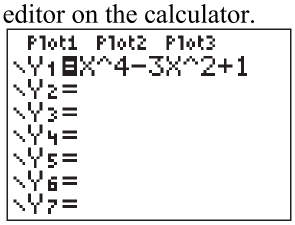
110. $y=(x+1)^3=x^3+3x^2+3x+1$

$$\frac{dy}{dx}=3x^2+6x+3$$

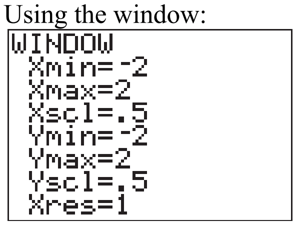
111. Answers will vary. Leibniz notation is more convenient when the variables are named to identify what they are. For example, marginal revenue using Leibniz notation is $\dfrac{dR}{dp}$. This notation allows the reader to determine that marginal revenue is the rate of change of revenue with respect to price. Thus Leibniz notation give the reader more information about the problem than the function notation $R'(p)$.

112. $f(x)=x^4-3x^2+1$

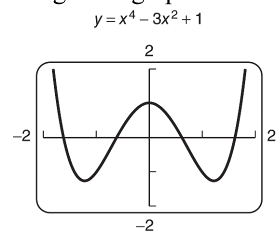
First we enter the equation into the graphing editor on the calculator.



Using the window:



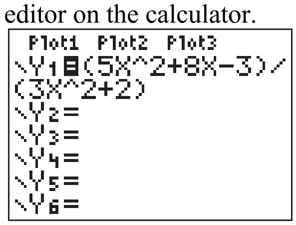
We get the graph:



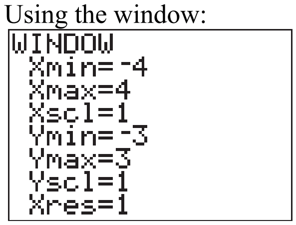
We estimate the x-values at which the tangent lines are horizontal are $x=-1.225,\ x=0,$ and, $x=1.225$.

113. $f(x)=\dfrac{5x^2+8x-3}{3x^2+2}$

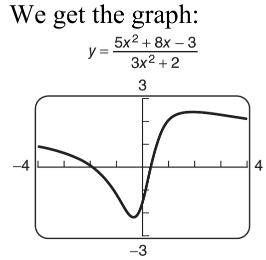
First we enter the equation into the graphing editor on the calculator.



Using the window:



We get the graph:

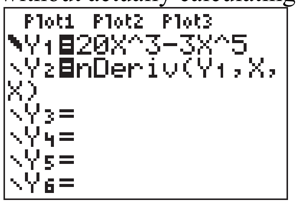


The horizontal tangents occur at the turning points of this function. Using the trace feature, or the minimum/maximum feature on the calculator, we find the turning points.

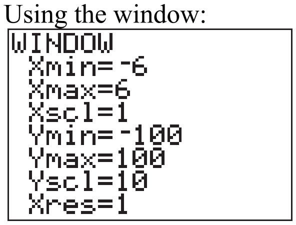
We estimate the x-values at which the tangent lines are horizontal are $x=-0.346$ and $x=1.929$.

114. $f(x)=20x^3-3x^5$

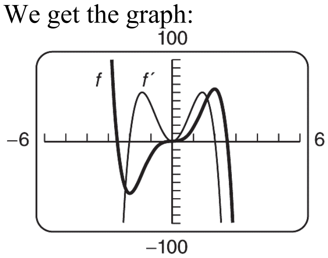
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:

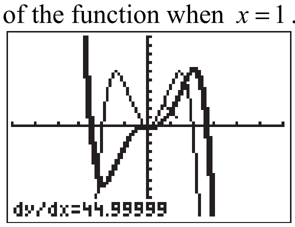


We get the graph:



Note, the function $f(x)$ is the thicker graph.

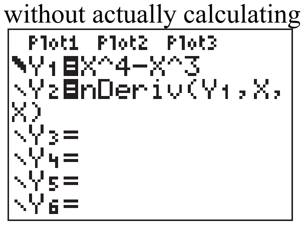
Using the calculator, we can find the derivative of the function when $x=1$.



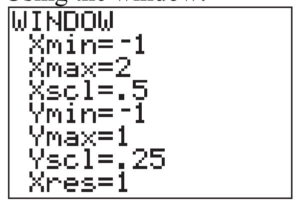
We have $f'(1)=45$.

115. $f(x)=x^4-x^3$

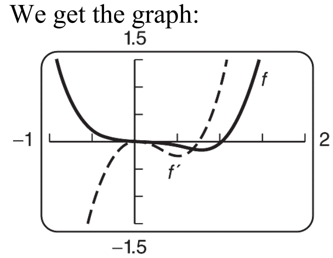
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:

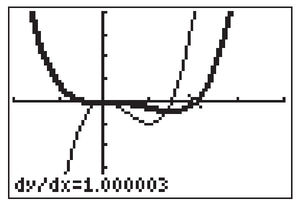


We get the graph:



Note, the function $f(x)$ is the solid graph.

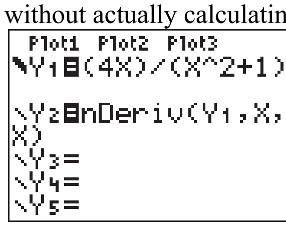
Using the calculator, we can find the derivative of the function when $x=1$.



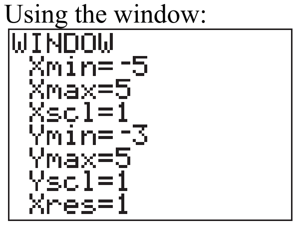
We have $f'(1)=1$.

116. $f(x)=\dfrac{4x}{x^2+1}$

Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

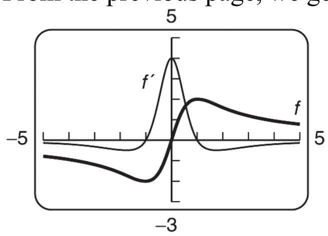


Using the window:



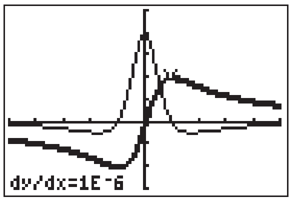
The solution is continued on the next page.

From the previous page, we get the graph:



Note, the function $f(x)$ is the thicker graph.

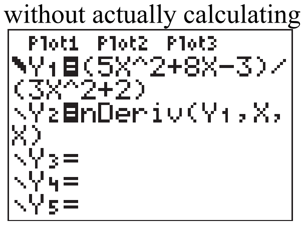
Using the calculator, we can find the derivative of the function when $x = 1$.



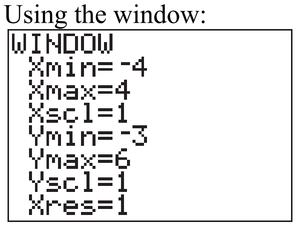
We have $f'(1) = 0$.

117. $f(x) = \dfrac{5x^2 + 8x - 3}{3x^2 + 2}$

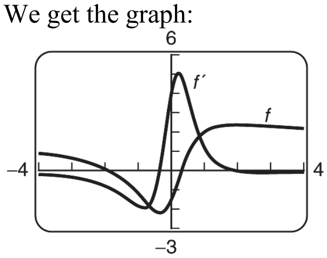
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:

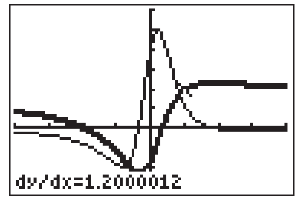


We get the graph:



Note, the function $f(x)$ is the thicker graph.

Using the calculator, we can find the derivative of the function when $x = 1$.



We have $f'(1) = 1.2$.

Exercise Set 1.6

1. Differentiate $y = x^9 \cdot x^4$ using the Product Rule (Theorem 5) .

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^9 \cdot x^4\right)$$
$$= x^9 \cdot \frac{d}{dx}\left(x^4\right) + x^4 \cdot \frac{d}{dx}\left(x^9\right)$$
$$= x^9 \cdot 4x^3 + x^4 \cdot 9x^8$$
$$= 4x^{12} + 9x^{12}$$
$$= 13x^{12}$$

Differentiate $y = x^9 \cdot x^4 = x^{13}$ using the Power Rule (Theorem 1).

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{13}\right)$$
$$= 13x^{13-1}$$
$$= 13x^{12}$$

The two results are equivalent.

2. Using the Product Rule:

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^5 \cdot x^6\right)$$
$$= x^5 \cdot 6x^5 + x^6 \cdot 5x^4$$
$$= 6x^{10} + 5x^{10}$$
$$= 11x^{10}$$

Using the Power Rule:

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^5 \cdot x^6\right)$$
$$= \frac{d}{dx} x^{11}$$
$$= 11x^{10}$$

The two results are equivalent.

3. Differentiate $f(x) = (2x+5)(3x-4)$ using the Product Rule (Theorem 5).

$$f'(x) = \frac{d}{dx}\left[(2x+5)(3x-4)\right]$$
$$= (2x+5)\cdot\frac{d}{dx}(3x-4) + (3x-4)\cdot\frac{d}{dx}(2x+5)$$
$$= (2x+5)\cdot 3 + (3x-4)\cdot 2$$
$$= 6x+15+6x-8$$
$$= 12x+7$$

Differentiate $f(x) = (2x+5)(3x-4)$ using the Power Rule (Theorem 1). First, we multiply the binomial terms in the function.

$$f(x) = (2x+5)(3x-4)$$
$$= 6x^2 + 7x - 20$$

Therefore, by Theorem 1, Theorem 2, and Theorem 4 we have:

$$f'(x) = \frac{d}{dx}\left(6x^2 + 7x - 20\right)$$
$$= \frac{d}{dx}\left(6x^2\right) + \frac{d}{dx}(7x) - \frac{d}{dx}(20) \quad \text{Theorem 4}$$
$$= 12x + 7 \quad \text{Theorem 1 and 2}$$

The two results are equivalent.

4. Using the Product Rule:

$$g'(x) = \frac{d}{dx}\left[(3x-2)(4x+1)\right]$$
$$= (3x-2)\cdot 4 + (4x+1)\cdot 3$$
$$= 12x - 8 + 12x + 3$$
$$= 24x - 5$$

Using the Power Rule:

$$g(x) = (3x-2)(4x+1) = 12x^2 - 5x - 2$$
$$g'(x) = \frac{d}{dx}\left(12x^2 - 5x - 2\right)$$
$$= 24x - 5$$

The two results are equivalent.

5. Differentiate $F(x) = 3x^4\left(x^2 - 4x\right)$ using the Product Rule.

$$F'(x) = \frac{d}{dx}\left[3x^4\left(x^2 - 4x\right)\right]$$
$$= 3x^4 \cdot \frac{d}{dx}\left(x^2 - 4x\right) + \left(x^2 - 4x\right)\cdot\frac{d}{dx}\left(3x^4\right)$$
$$= 3x^4 \cdot (2x-4) + \left(x^2 - 4x\right)\cdot 12x^3$$
$$= 6x^5 - 12x^4 + 12x^5 - 48x^4$$
$$= 18x^5 - 60x^4$$

Differentiate $F(x) = 3x^4\left(x^2 - 4x\right)$ using the Power Rule. First, we multiply the function.

$$F(x) = 3x^4\left(x^2 - 4x\right)$$
$$= 3x^6 - 12x^5$$

The solution is continued on the next page.

Therefore, we have:

$$F'(x)=\frac{d}{dx}\left(3x^6-12x^5\right)$$
$$=\frac{d}{dx}\left(3x^6\right)-\frac{d}{dx}\left(12x^5\right) \quad \text{Theorem 4}$$
$$=3\left(6x^5\right)-12\left(5x^4\right) \quad \text{Theorem 1, Theorem 3}$$
$$=18x^5-60x^4$$

The two results are equivalent.

6. Using the Product Rule:

$$G'(x)=\frac{d}{dx}\left[4x^2\left(x^3+5x\right)\right]$$
$$=4x^2\cdot\left(3x^2+5\right)+\left(x^3+5x\right)\cdot(8x)$$
$$=12x^4+20x^2+8x^4+40x^2$$
$$=20x^4+60x^2$$

Using the Power Rule:

$$G'(x)=\frac{d}{dx}\left(4x^5+20x^3\right)$$
$$=20x^4+60x^2$$

The two results are equivalent.

7. Differentiate $y=\left(3\sqrt{x}+2\right)x^2$ using the Product Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(3\sqrt{x}+2\right)x^2\right]$$
$$=\left(3\sqrt{x}+2\right)\cdot\frac{d}{dx}\left(x^2\right)+x^2\cdot\frac{d}{dx}\left(3\sqrt{x}+2\right)$$
$$=\left(3x^{1/2}+2\right)\cdot\frac{d}{dx}\left(x^2\right)+x^2\cdot\frac{d}{dx}\left(3x^{1/2}+2\right)$$
$$=\left(3x^{1/2}+2\right)\cdot 2x+x^2\cdot\frac{3}{2}x^{-1/2} \quad \text{Theorems 1, 2 and 4.}$$
$$=6x^{3/2}+4x+\frac{3}{2}x^{3/2}$$
$$=\frac{15}{2}x^{3/2}+4x$$

Differentiate $y=\left(3\sqrt{x}+2\right)x^2$ using the Power Rule. First, we multiply the function.

$$y=\left(3\sqrt{x}+2\right)x^2$$
$$=3x^{1/2+2}+2x^2$$
$$=3x^{5/2}+2x^2$$

Therefore, we have:

$$\frac{dy}{dx}=\frac{d}{dx}\left(3x^{5/2}+2x^2\right)$$
$$=\frac{d}{dx}\left(3x^{5/2}\right)+\frac{d}{dx}\left(2x^2\right) \quad \text{Theorem 4}$$
$$=3\left(\frac{5}{2}x^{5/2-1}\right)+2\left(2x^1\right) \quad \text{Theorem 1, Theorem 3}$$
$$=\frac{15}{2}x^{3/2}+4x$$

The two results are equivalent.

8. Using the Product Rule:

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(4\sqrt{x}+3\right)x^3\right]$$
$$=\left(4x^{1/2}+3\right)\cdot\frac{d}{dx}\left(x^3\right)+x^3\cdot\frac{d}{dx}\left(4x^{1/2}+3\right)$$
$$=\left(4x^{1/2}+3\right)\cdot 3x^2+x^3\cdot\frac{4}{2}x^{-1/2}$$
$$=14x^{5/2}+9x^2$$

Using the Power Rule.

$$y=\left(4\sqrt{x}+3\right)x^3=4x^{7/2}+3x^3$$

Therefore, we have:

$$\frac{dy}{dx}=\frac{d}{dx}\left(4x^{7/2}+3x^3\right)$$
$$=4\left(\frac{7}{2}x^{7/2-1}\right)+3\left(3x^2\right)$$
$$=14x^{5/2}+9x^2$$

The two results are equivalent.

9. Differentiate $f(x)=(2x+5)\left(3x^2-4x+1\right)$ using the Product Rule.

$$f'(x)=\frac{d}{dx}\left[(2x+5)\left(3x^2-4x+1\right)\right]$$
$$=(2x+5)\cdot\frac{d}{dx}\left(3x^2-4x+1\right)+\left(3x^2-4x+1\right)\cdot\frac{d}{dx}(2x+5)$$
$$=(2x+5)\cdot(6x-4)+\left(3x^2-4x+1\right)\cdot 2$$
$$=12x^2+22x-20+6x^2-8x+2$$
$$=18x^2+14x-18$$

Differentiate $f(x)=(2x+5)\left(3x^2-4x+1\right)$ using the Power Rule. First, we multiply the terms in the function.

$$f(x)=\left[(2x+5)\left(3x^2-4x+1\right)\right]$$
$$=6x^3+7x^2-18x+5$$

From the previous page, we have:

$$f'(x) = \frac{d}{dx}\left(6x^3 + 7x^2 - 18x + 5\right)$$
$$= \frac{d}{dx}\left(6x^3\right) + \frac{d}{dx}\left(7x^2\right) - \frac{d}{dx}\left(18x\right) + \frac{d}{dx}(5)$$
$$= 18x^2 + 14x - 18$$

The two results are equivalent.

10. Using the Product Rule:

$$g'(x) = \frac{d}{dx}\left[(4x-3)\left(2x^2+3x+5\right)\right]$$
$$= (4x-3)\cdot(4x+3) + \left(2x^2+3x+5\right)\cdot 4$$
$$= 16x^2 - 9 + 8x^2 + 12x + 20$$
$$= 24x^2 + 12x + 11$$

Using the Power Rule:

$$g(x) = (4x-3)\left(2x^2+3x+5\right)$$
$$= 8x^3 + 6x^2 + 11x - 15$$
$$g'(x) = \frac{d}{dx}\left(8x^3 + 6x^2 + 11x - 15\right)$$
$$= 24x^2 + 12x + 11$$

The two results are equivalent.

11. Differentiate $F(t) = \left(\sqrt{t}+2\right)\left(3t - 4\sqrt{t} + 7\right)$ using the Product Rule.

$$F'(t) = \frac{d}{dt}\left[\left(\sqrt{t}+2\right)\left(3t-4\sqrt{t}+7\right)\right]$$
$$= \left(t^{1/2}+2\right)\cdot\frac{d}{dt}\left(3t - 4t^{1/2} + 7\right) + \left(3t - 4t^{1/2} + 7\right)\cdot\frac{d}{dt}\left(t^{1/2}+2\right) \qquad \left[\sqrt{t} = t^{1/2}\right]$$
$$= \left(t^{1/2}+2\right)\cdot\left(3 - 4\left(\frac{1}{2}t^{-1/2}\right)\right) + \left(3t - 4t^{1/2}+7\right)\cdot\left(\frac{1}{2}t^{-1/2}\right)$$

The solution is continued at the top of the next column.

$$F'(t) = \left(t^{1/2}+2\right)\cdot\left(3 - 2t^{-1/2}\right) + \left(3t - 4t^{1/2}+7\right)\cdot\left(\frac{1}{2}t^{-1/2}\right)$$
$$= 3t^{1/2} - 2 + 6 - 4t^{-1/2} + \frac{3}{2}t^{1/2} - 2 + \frac{7}{2}t^{-1/2}$$
$$= \frac{9}{2}t^{1/2} - \frac{1}{2}t^{-1/2} + 2$$
$$= \frac{9\sqrt{t}}{2} - \frac{1}{2\sqrt{t}} + 2$$

Differentiate $F(t) = \left(\sqrt{t}+2\right)\left(3t - 4\sqrt{t} + 7\right)$ using the Power Rule

$$F(t) = \left(\sqrt{t}+2\right)\left(3t-4\sqrt{t}+7\right)$$
$$= 3t^{3/2} - 4t + 7t^{1/2} + 6t - 8t^{1/2} + 14$$
$$= 3t^{3/2} - t^{1/2} + 2t + 14$$

Therefore, we have:

$$F'(t) = \frac{d}{dt}\left(3t^{3/2} - t^{1/2} + 2t + 14\right)$$
$$= \frac{d}{dt}\left(3t^{3/2}\right) - \frac{d}{dt}\left(t^{1/2}\right) + \frac{d}{dt}(2t) + \frac{d}{dt}(14)$$
$$= \frac{9t^{1/2}}{2} - \frac{1}{2}t^{-1/2} + 2$$
$$= \frac{9\sqrt{t}}{2} - \frac{1}{2\sqrt{t}} + 2$$

The two results are equivalent.

12. Using the Product Rule:

$$G'(t) = \frac{d}{dt}\left[\left(2t + 3\sqrt{t} + 5\right)\left(\sqrt{t}+4\right)\right]$$
$$= \left(2t + 3t^{1/2} + 5\right)\cdot\left(\frac{1}{2}t^{-1/2}\right) + \left(t^{1/2}+4\right)\cdot\left(2 + \frac{3}{2}t^{-1/2}\right)$$
$$= t^{1/2} + \frac{3}{2} + \frac{5}{2}t^{-1/2} + 2t^{1/2} + \frac{3}{2} + 8 + 6t^{-1/2}$$
$$= 3t^{1/2} + \frac{17}{2}t^{-1/2} + 11$$
$$= 3\sqrt{t} + \frac{17}{2\sqrt{t}} + 11$$

The solution is continued on the next page.

Using the Power Rule:

$$G(x) = \left(2t + 3\sqrt{t} + 5\right)\left(\sqrt{t} + 4\right)$$
$$= 2t^{3/2} + 17t^{1/2} + 11t + 20$$
$$G'(t) = \frac{d}{dt}\left(2t^{3/2} + 17t^{1/2} + 11t + 20\right)$$
$$= 3t^{1/2} + \frac{17}{2}t^{-1/2} + 11$$
$$= 3\sqrt{t} + \frac{17}{2\sqrt{t}} + 11$$

The two results are equivalent.

13. Differentiate $y = \dfrac{x^6}{x^4}$ using the Quotient Rule (Theorem 6).

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^6}{x^4}\right)$$
$$= \frac{x^4 \frac{d}{dx}\left(x^6\right) - x^6 \frac{d}{dx}\left(x^4\right)}{\left(x^4\right)^2}$$
$$= \frac{x^4\left(6x^5\right) - x^6\left(4x^3\right)}{x^8}$$
$$\frac{dy}{dx} = \frac{6x^9 - 4x^9}{x^8}$$
$$= \frac{2x^9}{x^8}$$
$$= 2x, \qquad \text{for } x \neq 0$$

Differentiate $y = \dfrac{x^6}{x^4} = x^2$ using the Power Rule.

$$\frac{dy}{dx} = \frac{d}{dx}x^2$$
$$= 2x^{2-1}$$
$$= 2x, \qquad \text{for } x \neq 0$$

The two results are equivalent.

14. Using the Quotient Rule.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^7}{x^3}\right)$$
$$= \frac{x^3 \frac{d}{dx}\left(x^7\right) - x^7 \frac{d}{dx}\left(x^3\right)}{\left(x^3\right)^2}$$
$$= \frac{x^3\left(7x^6\right) - x^7\left(3x^2\right)}{x^6}$$

The solution is continued on the next column.

$$\frac{dy}{dx} = \frac{7x^9 - 3x^9}{x^6}$$
$$= \frac{4x^9}{x^6}$$
$$= 4x^3, \qquad \text{for } x \neq 0$$

Using the Power Rule.

$$y = \frac{x^7}{x^3} = x^4$$
$$\frac{dy}{dx} = \frac{d}{dx}x^4$$
$$= 4x^3, \quad \text{for } x \neq 0$$

The two results are equivalent.

15. Differentiate $g(x) = \dfrac{3x^7 - x^3}{x}$ using the Quotient Rule.

$$g'(x) = \frac{d}{dx}\left(\frac{3x^7 - x^3}{x}\right)$$
$$= \frac{x\frac{d}{dx}\left(3x^7 - x^3\right) - \left(3x^7 - x^3\right)\frac{d}{dx}(x)}{(x)^2}$$
$$= \frac{x\left(21x^6 - 3x^2\right) - \left(3x^7 - x^3\right)(1)}{x^2}$$
$$= \frac{18x^7 - 2x^3}{x^2}$$
$$= \frac{x^2\left(18x^5 - 2x\right)}{x^2}$$
$$= 18x^5 - 2x, \qquad \text{for } x \neq 0$$

Differentiate $g(x) = \dfrac{3x^7 - x^3}{x}$ using the Power Rule. First, factor the numerator and divide the common factors.

$$g(x) = \frac{3x^7 - x^3}{x}$$
$$= \frac{x\left(3x^6 - x^2\right)}{x}$$
$$= 3x^6 - x^2$$
$$g'(x) = \frac{d}{dx}\left(3x^6 - x^2\right)$$
$$= 18x^5 - 2x, \qquad \text{for } x \neq 0$$

The two results are equivalent.

16. Using the Quotient Rule.

$$f'(x) = \frac{d}{dx}\left(\frac{2x^5 + x^2}{x}\right)$$
$$= \frac{x(10x^4 + 2x) - (2x^5 + x^2)(1)}{x^2}$$
$$= \frac{10x^5 + 2x^2 - 2x^5 - x^2}{x^2}$$
$$= \frac{8x^5 + x^2}{x^2}$$
$$= \frac{x^2(8x^3 + 1)}{x^2}$$
$$= 8x^3 + 1, \quad \text{for } x \neq 0$$

Using the Power Rule.

$$f(x) = \frac{2x^5 + x^2}{x} = \frac{x(2x^4 + x)}{x} = 2x^4 + x$$
$$f'(x) = \frac{d}{dx}(2x^4 + x)$$
$$= 8x^3 + 1, \quad \text{for } x \neq 0$$

The two results are equivalent.

17. Differentiate $G(x) = \frac{8x^3 - 1}{2x - 1}$ using the Quotient Rule.

$$G'(x) = \frac{d}{dx}\left(\frac{8x^3 - 1}{2x - 1}\right)$$
$$= \frac{(2x-1)\frac{d}{dx}(8x^3 - 1) - (8x^3 - 1)\frac{d}{dx}(2x - 1)}{(2x-1)^2}$$
$$= \frac{(2x-1)(24x^2) - (8x^3 - 1)(2)}{(2x-1)^2}$$
$$= \frac{(2x-1)(24x^2) - (2x-1)(4x^2 + 2x + 1)(2)}{(2x-1)^2}$$
$$= \frac{(2x-1)\left[(24x^2) - (4x^2 + 2x + 1)(2)\right]}{(2x-1)^2}$$
$$= \frac{(2x-1)\left[(24x^2) - (8x^2 + 4x + 2)\right]}{(2x-1)^2}$$
$$= \frac{\left[16x^2 - 4x - 2\right]}{(2x-1)}$$
$$= \frac{(2x-1)(8x+2)}{(2x-1)}$$
$$= 8x + 2; \quad x \neq \tfrac{1}{2}$$

Differentiate $G(x) = \frac{8x^3 - 1}{2x - 1}$ using the Power Rule.
First, factor the numerator and divide the common factors.

$$G(x) = \frac{8x^3 - 1}{2x - 1}$$
$$= \frac{(2x-1)(4x^2 + 2x + 1)}{2x - 1} \quad \text{Difference of cubes}$$
$$= 4x^2 + 2x + 1$$
$$G'(x) = \frac{d}{dx}(4x^2 + 2x + 1)$$
$$= 8x + 2; \quad x \neq \tfrac{1}{2}$$

The two results are equivalent.

18. Using the Quotient Rule.

$$F'(x)=\frac{d}{dx}\left(\frac{x^3+27}{x+3}\right)$$
$$=\frac{(x+3)(3x^2)-(x^3+27)(1)}{(x+3)^2}$$
$$=\frac{(x+3)(3x^2)-(x+3)(x^2-3x+9)}{(x+3)^2}$$
$$=\frac{(x+3)\left(3x^2-(x^2-3x+9)\right)}{(x+3)^2}$$
$$=\frac{(2x^2+3x-9)}{(x+3)}$$
$$=\frac{(x+3)(2x-3)}{(x+3)}$$
$$=2x-3, \qquad \text{for } x\neq -3$$

Using the Power Rule.

$$F(x)=\frac{x^3+27}{x+3}$$
$$=\frac{(x+3)(x^2-3x+9)}{x+3}$$
$$=x^2-3x+9$$
$$F'(x)=\frac{d}{dx}(x^2-3x+9)$$
$$=2x-3, \qquad \text{for } x\neq -3$$

The two results are equivalent.

19. Differentiate $y=\frac{t^2-16}{t+4}$ using the Quotient Rule.

$$\frac{dy}{dt}=\frac{d}{dt}\left(\frac{t^2-16}{t+4}\right)$$
$$=\frac{(t+4)\frac{d}{dt}(t^2-16)-(t^2-16)\frac{d}{dt}(t+4)}{(t+4)^2}$$
$$=\frac{(t+4)(2t)-(t^2-16)(1)}{(t+4)^2}$$
$$=\frac{2t^2+8t-t^2+16}{(t+4)^2}$$
$$=\frac{t^2+8t+16}{(t+4)^2}$$

The derivative is simplified at the top of the next column.

$$\frac{dy}{dt}=\frac{(t+4)^2}{(t+4)^2}$$
$$=1; \qquad t\neq -4$$

Differentiate $y=\frac{t^2-16}{t+4}$ using the Power Rule.

First, factor the numerator and divide the common factors.

$$y=\frac{t^2-16}{t+4}$$
$$=\frac{(t+4)(t-4)}{t+4} \qquad \text{Difference of squares}$$
$$=t-4$$
$$\frac{dy}{dt}=\frac{d}{dt}(x-4)$$
$$=1, \qquad \text{for } t\neq -4$$

The two results are equivalent.

20. Using the Quotient Rule.

$$\frac{dy}{dt}=\frac{d}{dt}\left(\frac{t^2-25}{t-5}\right)$$
$$=\frac{(t-5)(2t)-(t^2-25)(1)}{(t-5)^2}$$
$$=\frac{2t^2-10t-(t^2-25)}{(t-5)^2}$$
$$=\frac{t^2-10t+25}{(t-5)^2}$$
$$=\frac{(t-5)^2}{(t-5)^2}$$
$$=1, \qquad \text{for } t\neq 5$$

Using the Power Rule.

$$y=\frac{t^2-25}{t-5}$$
$$=\frac{(t-5)(t+5)}{t-5}$$
$$=t+5$$
$$\frac{dy}{dt}=\frac{d}{dt}(t+5)$$
$$=1, \qquad \text{for } t\neq 5$$

The two results are equivalent.

21. $g(x)=(5x^2+4x-3)(2x^2-3x+1)$

Using the Product Rule, we have:

$$g'(x)=\frac{d}{dx}\left[(5x^2+4x-3)(2x^2-3x+1)\right]$$
$$=(5x^2+4x-3)\cdot\frac{d}{dx}(2x^2-3x+1)+$$
$$(2x^2-3x+1)\cdot\frac{d}{dx}(5x^2+4x-3)$$
$$=(5x^2+4x-3)\cdot(4x-3)+$$
$$(2x^2-3x+1)\cdot(10x+4)$$

Simplifying we get

$$=(20x^3+x^2-24x+9)+$$
$$(20x^3-22x^2-2x+4)$$
$$=40x^3-21x^2-26x+13$$

22. $f(x)=(3x^2-2x+5)(4x^2+3x-1)$

Using the Product Rule, we have:

$$f'(x)=\frac{d}{dx}\left[(3x^2-2x+5)(4x^2+3x-1)\right]$$
$$=(3x^2-2x+5)\cdot\frac{d}{dx}(4x^2+3x-1)+$$
$$(4x^2+3x-1)\cdot\frac{d}{dx}(3x^2-2x+5)$$
$$=(3x^2-2x+5)\cdot(8x+3)+$$
$$(4x^2+3x-1)\cdot(6x-2)$$

Simplifying, we get

$$=(24x^3-7x^2+34x+15)+$$
$$(24x^3+10x^2-12x+2)$$
$$=48x^3+3x^2+22x+17$$

23. $y=\frac{5x^2-1}{2x^3+3}$

Using the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{5x^2-1}{2x^3+3}\right)$$
$$=\frac{(2x^3+3)\frac{d}{dx}(5x^2-1)-(5x^2-1)\frac{d}{dx}(2x^3+3)}{(2x^3+3)^2}$$
$$=\frac{(2x^3+3)(10x)-(5x^2-1)(6x^2)}{(2x^3+3)^2}$$
$$=\frac{20x^4+30x-30x^4+6x^2}{(2x^3+3)^2}$$
$$=\frac{-10x^4+6x^2+30x}{(2x^3+3)^2}$$
$$=\frac{-2x(5x^3-3x-15)}{(2x^3+3)^2}$$

24. $y=\frac{3x^4+2x}{x^3-1}$

Using the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{3x^4+2x}{x^3-1}\right)$$
$$=\frac{(x^3-1)\frac{d}{dx}(3x^4+2x)-(3x^4+2x)\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$
$$=\frac{(x^3-1)(12x^3+2)-(3x^4+2x)(3x^2)}{(x^3-1)^2}$$
$$=\frac{12x^6-10x^3-2-(9x^6+6x^3)}{(x^3-1)^2}$$
$$=\frac{3x^6-16x^3-2}{(x^3-1)^2}$$

25. $F(x)=\left(-3x^2+4x\right)\left(7\sqrt{x}+1\right)$

$F(x)=\left(-3x^2+4x\right)\left(7x^{1/2}+1\right) \qquad \left[\sqrt{x}=x^{1/2}\right]$

Using the Product Rule, we have:

$$F(x)=\frac{d}{dx}\left[\left(-3x^2+4x\right)\left(7x^{1/2}+1\right)\right]$$
$$=\left(-3x^2+4x\right)\cdot\frac{d}{dx}\left(7x^{1/2}+1\right)+\left(7x^{1/2}+1\right)\cdot\frac{d}{dx}\left(-3x^2+4x\right)$$
$$=\left(-3x^2+4x\right)\cdot\left(\frac{7}{2}x^{-1/2}\right)+\left(7x^{1/2}+1\right)\cdot(-6x+4)$$

Simplifying we get

$$=\left(-\frac{21}{2}x^{3/2}+14x^{1/2}\right)+\left(-42x^{3/2}-6x+28x^{1/2}+4\right)$$
$$=-\frac{105}{2}x^{3/2}-6x+42x^{1/2}+4$$

26. $G(x)=\left(8x+\sqrt{x}\right)\left(5x^2+3\right)$

$G(x)=\left(8x+x^{1/2}\right)\left(5x^2+3\right) \qquad \left[\sqrt{x}=x^{1/2}\right]$

Using the Product Rule, we have:

$$G'(x)=\frac{d}{dx}\left[\left(8x+x^{1/2}\right)\left(5x^2+3\right)\right]$$
$$=\left(8x+x^{1/2}\right)\cdot\frac{d}{dx}\left(5x^2+3\right)+\left(5x^2+3\right)\cdot\frac{d}{dx}\left(8x+x^{1/2}\right)$$
$$=\left(8x+x^{1/2}\right)\cdot(10x)+\left(5x^2+3\right)\cdot\left(8+\frac{1}{2}x^{-1/2}\right)$$

Simplifying, we get

$$=\left(80x^2+10x^{3/2}\right)+\left(40x^2+\frac{5}{2}x^{3/2}+\frac{3}{2}x^{-1/2}+24\right)$$
$$=120x^2+\frac{25}{2}x^{3/2}+\frac{3}{2}x^{-1/2}+24$$

27. $g(t)=\dfrac{t}{3-t}+5t^3$

Differentiating we have:

$$g'(t)=\frac{d}{dt}\left(\frac{t}{3-t}+5t^3\right)$$
$$=\frac{d}{dt}\left(\frac{t}{3-t}\right)+\frac{d}{dt}\left(5t^3\right)$$

Using the derivative, we will apply the Quotient Rule to the first term, and the Power Rule to the second term.

$$g'(t)=\underbrace{\frac{(3-t)\cdot\frac{d}{dt}(t)-t\cdot\frac{d}{dt}(3-t)}{(3-t)^2}}_{\text{Quotient Rule}}+15t^2$$
$$=\frac{(3-t)(1)-t(-1)}{(3-t)^2}+15t^2$$
$$=\frac{3}{(3-t)^2}+15t^2$$

28. $f(t)=\dfrac{t}{5+2t}-2t^4$

$$f'(t)=\frac{d}{dt}\left(\frac{t}{5+2t}-2t^4\right)$$
$$=\underbrace{\frac{(5+2t)(1)-t(2)}{(5+2t)^2}}_{\text{Quotient Rule}}-8t^3$$
$$f'(t)=\frac{5}{(5+2t)^2}-8t^3$$

29. $G(x)=(5x-4)^2=(5x-4)(5x-4)$

Using the Product Rule, we have

$$G'(x)=\frac{d}{dx}\left[(5x-4)(5x-4)\right]$$
$$=(5x-4)\cdot\frac{d}{dx}(5x-4)+(5x-4)\cdot\frac{d}{dx}(5x-4)$$
$$=(5x-4)\cdot\frac{d}{dx}(5x-4)+(5x-4)\cdot\frac{d}{dx}(5x-4)$$
$$=(5x-4)\cdot(5)+(5x-4)\cdot(5)$$
$$=50x-40$$

30. $F(x)=(x+3)^2=(x+3)(x+3)$

Using the Product Rule, we have

$$F'(x)=\frac{d}{dx}\left[(x+3)(x+3)\right]$$
$$=(x+3)\cdot\frac{d}{dx}(x+3)+(x+3)\cdot\frac{d}{dx}(x+3)$$
$$=(x+3)\cdot(1)+(x+3)\cdot(1)$$
$$=2x+6$$
$$=2(x+3)$$

31. $y=\left(x^3-4x\right)^2=\left(x^3-4x\right)\left(x^3-4x\right)$

Using the Product Rule, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(x^3-4x\right)\left(x^3-4x\right)\right]$$
$$=\left(x^3-4x\right)\cdot\frac{d}{dx}\left(x^3-4x\right)+\left(x^3-4x\right)\cdot\frac{d}{dx}\left(x^3-4x\right)$$
$$=\left(x^3-4x\right)\cdot\left(3x^2-4\right)+\left(x^3-4x\right)\cdot\left(3x^2-4\right)$$

Simplifying, we get

$$=2\left(x^3-4x\right)\left(3x^2-4\right)$$
$$=2x\left(x^2-4\right)\left(3x^2-4\right)$$

32. $y=\left(3x^2-4x+5\right)^2$

$$=\left(3x^2-4x+5\right)\left(3x^2-4x+5\right)$$

Using the Product Rule, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(3x^2-4x+5\right)\left(3x^2-4x+5\right)\right]$$
$$=\left(3x^2-4x+5\right)\cdot\frac{d}{dx}\left(3x^2-4x+5\right)+\left(3x^2-4x+5\right)\cdot\frac{d}{dx}\left(3x^2-4x+5\right)$$
$$=\left(3x^2-4x+5\right)\cdot(6x-4)+\left(3x^2-4x+5\right)\cdot(6x-4)$$
$$=2\left(3x^2-4x+5\right)(6x-4)$$
$$=4(3x-2)\left(3x^2-4x+5\right)$$

33. $f(x)=6x^{-4}\left(6x^3+10x^2-8x+3\right)$

Using the Product Rule:

$$f'(x)=\left(6x^{-4}\right)\frac{d}{dx}\left(6x^3+10x^2-8x+3\right)+\left(6x^3+10x^2-8x+3\right)\frac{d}{dx}\left(6x^{-4}\right)$$
$$=\left(6x^{-4}\right)\left(18x^2+20x-8\right)+\left(6x^3+10x^2-8x+3\right)\left(-24x^{-5}\right)$$

Simplifying, we get

$$=108x^{-2}+120x^{-3}-48x^{-4}-144x^{-2}-240x^{-3}+192x^{-4}-72x^{-5}$$
$$=-36x^{-2}-120x^{-3}+144x^{-4}-72x^{-5}$$

34. $g(x)=5x^{-3}\left(x^4-5x^3+10x-2\right)$

Using the Product Rule:

$$g'(x)=\left(5x^{-3}\right)\left(4x^3-15x^2+10\right)+\left(x^4-5x^3+10x-2\right)\left(-15x^{-4}\right)$$
$$=20-75x^{-1}+50x^{-3}-15+75x^{-1}-150x^{-3}+30x^{-4}$$
$$=5-100x^{-3}+30x^{-4}$$

35. $F(t)=\left(t+\frac{2}{t}\right)\left(t^2-3\right)=\left(t+2t^{-1}\right)\left(t^2-3\right)$

Using the Product Rule, we have:

$$F'(t)=\frac{d}{dt}\left[\left(t+2t^{-1}\right)\left(t^2-3\right)\right]$$
$$=\left(t+2t^{-1}\right)\cdot\frac{d}{dt}\left(t^2-3\right)+\left(t^2-3\right)\cdot\frac{d}{dt}\left(t+2t^{-1}\right)$$
$$=\left(t+2t^{-1}\right)\cdot(2t)+\left(t^2-3\right)\cdot\left(1-2t^{-2}\right)$$

Simplifying, we get

$$=2t^2+4+\left(t^2-2-3+6t^{-2}\right)$$
$$=3t^2-1+6t^{-2}$$
$$=3t^2-1+\frac{6}{t^2}$$

36. $G(t)=\left(3t^5-t^2\right)\left(t-\frac{5}{t}\right)=\left(3t^5-t^2\right)\left(t-5t^{-1}\right)$

Using the Product Rule, we have:

$$G'(t)=\frac{d}{dt}\left[\left(3t^5-t^2\right)\left(t-5t^{-1}\right)\right]$$
$$=\left(3t^5-t^2\right)\left(1+5t^{-2}\right)+\left(t-5t^{-1}\right)\left(15t^4-2t\right)$$

Simplifying, we get

$$=\left(3t^5+15t^3-t^2-5\right)+\left(15t^5-75t^3-2t^2+10\right)$$
$$=18t^5-60t^3-3t^2+5, \quad \text{for } t\neq 0$$

37. $y=\frac{x^3-1}{x^2+1}+4x^3$

Differentiating we have:

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^3-1}{x^2+1}+4x^3\right)$$
$$=\frac{d}{dx}\left(\frac{x^3-1}{x^2+1}\right)+\frac{d}{dx}\left(4x^3\right)$$

We will apply the Quotient Rule to the first term, and the Power Rule to the second term.

$$\frac{dy}{dx}=\underbrace{\frac{\left(x^2+1\right)\cdot\left(3x^2\right)-\left(x^3-1\right)\cdot(2x)}{\left(x^2+1\right)^2}}_{\text{Quotient Rule}}+12x$$

Simplifying, we get

$$=\frac{\left(3x^4+3x^2\right)-\left(2x^4-2x\right)}{\left(x^2+1\right)^2}+12x^2$$
$$=\frac{x^4+3x^2+2x}{\left(x^2+1\right)^2}+12x^2$$

38. $y=\frac{x^2+1}{x^3-1}-5x^2$

Differentiating we have:

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2+1}{x^3-1}-5x^2\right)$$
$$=\frac{d}{dx}\left(\frac{x^2+1}{x^3-1}\right)-\frac{d}{dx}\left(5x^2\right)$$

We will apply the Quotient Rule to the first term, and the Power Rule to the second term.

$$\frac{dy}{dx}=\underbrace{\frac{\left(x^3-1\right)\cdot(2x)-\left(x^2+1\right)\cdot\left(3x^2\right)}{\left(x^3-1\right)^2}}_{\text{Quotient Rule}}-10x$$

Simplifying, we get

$$=\frac{2x^4-2x-\left(3x^4+3x^2\right)}{\left(x^3-1\right)^2}-10x$$
$$=\frac{-x^4-3x^2-2x}{\left(x^3-1\right)^2}-10x$$

39. $y=\frac{\sqrt[3]{x}-7}{\sqrt{x}+3}=\frac{x^{1/3}-7}{x^{1/2}+3}$

Using the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^{1/3}-7}{x^{1/2}+3}\right)$$
$$=\frac{\left(x^{1/2}+3\right)\frac{d}{dx}\left(x^{1/3}-7\right)-\left(x^{1/3}-7\right)\frac{d}{dx}\left(x^{1/2}+3\right)}{\left(x^{1/2}+3\right)^2}$$
$$=\frac{\left(x^{1/2}+3\right)\left(\frac{1}{3}x^{-2/3}\right)-\left(x^{1/3}-7\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}+3\right)^2}$$

Note, the previous derivative can be simplified as follows

$$\frac{dy}{dx}=\frac{\left(x^{1/2}+3\right)\left(\frac{1}{3}x^{-2/3}\right)-\left(x^{1/3}-7\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}+3\right)^2}$$
$$=\frac{\frac{1}{3}x^{-1/6}+x^{-2/3}-\frac{1}{2}x^{-1/6}+\frac{7}{2}x^{-1/2}}{\left(x^{1/2}+3\right)^2}$$
$$=\frac{x^{-2/3}-\frac{1}{6}x^{-1/6}+\frac{7}{2}x^{-1/2}}{\left(x^{1/2}+3\right)^2}\cdot\frac{6x^{2/3}}{6x^{2/3}}$$
$$=\frac{6-\sqrt{x}+21x^{1/6}}{6x^{2/3}\left(\sqrt{x}+3\right)^2}$$

40. $y=\frac{\sqrt{x}+4}{\sqrt[3]{x}-5}=\frac{x^{1/2}+4}{x^{1/3}-5}$

Using the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^{1/2}+4}{x^{1/3}-5}\right)$$

$$=\frac{\left(x^{1/3}-5\right)\left(\frac{1}{2}x^{-1/2}\right)-\left(x^{1/2}+4\right)\left(\frac{1}{3}x^{-2/3}\right)}{\left(x^{1/3}-5\right)^2}$$

Simplifying the derivative, we get

$$\frac{dy}{dx}==\frac{-8+\sqrt{x}-15x^{1/6}}{6x^{2/3}\left(x^{1/3}-5\right)^2}$$

41. $f(x)=\frac{x^{-1}}{x+x^{-1}}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{x^{-1}}{x+x^{-1}}\right)$$

$$=\frac{\left(x+x^{-1}\right)\left(-1x^{-2}\right)-\left(x^{-1}\right)\left(1-x^{-2}\right)}{\left(x+x^{-1}\right)^2}$$

$$=\frac{-x^{-1}-x^{-3}-x^{-1}+x^{-3}}{\left(x^{-1}+x\right)^2}$$

$$=\frac{-2x^{-1}}{\left(x^{-1}+x\right)^2},\qquad \text{for } x\neq 0$$

The previous derivative can be simplified as follows:

$$f'(x)=\frac{-2x^{-1}}{\left(x^{-1}+x\right)^2}$$

$$=\frac{-\frac{2}{x}}{\left(\frac{1}{x}+x\right)^2}$$

$$=\frac{-\frac{2}{x}}{\left(\frac{1+x^2}{x}\right)^2}$$

$$=\frac{-2}{x}\cdot\frac{x^2}{\left(1+x^2\right)^2}$$

$$=\frac{-2x}{\left(x^2+1\right)^2},\qquad \text{for } x\neq 0$$

42. $f(x)=\frac{x}{x^{-1}+1}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{x}{x^{-1}+1}\right)$$

$$=\frac{\left(x^{-1}+1\right)\frac{d}{dx}(x)-(x)\frac{d}{dx}\left(x^{-1}+1\right)}{\left(x^{-1}+1\right)^2}$$

$$=\frac{\left(x^{-1}+1\right)(1)-(x)\left(-1x^{-2}\right)}{\left(x^{-1}+1\right)^2}$$

$$=\frac{x^{-1}+1+x^{-1}}{\left(x^{-1}+1\right)^2}$$

$$=\frac{2x^{-1}+1}{\left(x^{-1}+1\right)^2},\qquad \text{for } x\neq 0 \text{ and } x\neq -1$$

Note, the previous derivative could be simplified as follows:

$$f'(x)=\frac{x(x+2)}{(1+x)^2},\qquad \text{for } x\neq 0 \text{ and } x\neq -1$$

43. $F(t)=\frac{1}{t-4}$

Using the Quotient Rule, we have

$$F'(t)=\frac{d}{dt}\left(\frac{1}{t-4}\right)$$

$$=\frac{(t-4)\frac{d}{dt}(1)-(1)\frac{d}{dt}(t-4)}{(t-4)^2}$$

$$=\frac{(t-4)(0)-(1)(1)}{(t-4)^2}$$

$$=\frac{-1}{(t-4)^2}$$

44. $G(t)=\frac{1}{t+2}$

Using the Quotient Rule, we have

$$G'(t)=\frac{d}{dt}\left(\frac{1}{t+2}\right)$$

$$=\frac{(t+2)(0)-(1)(1)}{(t+2)^2}$$

$$=\frac{-1}{(t+2)^2}$$

45. $f(x)=\dfrac{3x^2-5x}{x^2-1}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{3x^2-5x}{x^2-1}\right)$$
$$=\frac{(x^2-1)\frac{d}{dx}(3x^2-5x)-(3x^2-5x)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$
$$=\frac{(x^2-1)(6x-5)-(3x^2-5x)(2x)}{(x^2-1)^2}$$
$$=\frac{6x^3-5x^2-6x+5-(6x^3-10x^2)}{(x^2-1)^2}$$
$$=\frac{5x^2-6x+5}{(x^2-1)^2}$$

46. $f(x)=\dfrac{3x^2+2x}{x^2+1}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{3x^2+2x}{x^2+1}\right)$$
$$=\frac{(x^2+1)(6x+2)-(3x^2+2x)(2x)}{(x^2+1)^2}$$
$$=\frac{6x^3+2x^2+6x+2-(6x^3+4x^2)}{(x^2+1)^2}$$
$$=\frac{-2x^2+6x+2}{(x^2+1)^2}$$
$$=\frac{-2(x^2-3x-1)}{(x^2+1)^2}$$

47. $g(t)=\dfrac{-t^2+3t+5}{t^2-2t+4}$

Using the Quotient Rule, we have

$$g'(t)=\frac{d}{dt}\left(\frac{-t^2+3t+5}{t^2-2t+4}\right)$$
$$=\frac{(t^2-2t+4)\frac{d}{dt}(-t^2+3t+5)}{(t^2-2t+4)^2}-$$
$$\frac{(-t^2+3t+5)\frac{d}{dt}(t^2-2t+4)}{(t^2-2t+4)^2}$$
$$=\frac{(t^2-2t+4)(-2t+3)-(-t^2+3t+5)(2t-2)}{(t^2-2t+4)^2}$$

Note, the previous derivative could be simplified as follows:

$$g'(t)=\frac{-2t^3+7t^2-14t+12-(-2t^3+8t^2+4t-10)}{(t^2-2t+4)^2}$$
$$=\frac{-t^2-18t+22}{(t^2-2t+4)^2}$$

48. $f(t)=\dfrac{3t^2+2t-1}{-t^2+4t+1}$

Using the Quotient Rule, we have

$$f'(t)=\frac{d}{dt}\left(\frac{3t^2+2t-1}{-t^2+4t+1}\right)$$
$$=\frac{(-t^2+4t+1)(6t+2)-(3t^2+2t-1)(-2t+4)}{(-t^2+4t+1)^2}$$

The previous derivative can be simplified as follows:

$$f'(t)=\frac{14t^2+4t+6}{(t^2-4t-1)^2}.$$

49. $y = \dfrac{8}{x^2+4}$

$$\frac{dy}{dx} = \frac{(x^2+4)(0) - 8(2x)}{(x^2+4)^2}$$

$$\frac{dy}{dx} = \frac{-16x}{(x^2+4)^2}$$

a) When $x = 0$, $\dfrac{dy}{dx} = \dfrac{-16(0)}{(0^2+4)^2} = 0$, so the slope of the tangent line at $(0,2)$ is 0. The equation of the horizontal line passing through $(0,2)$ is $y = 2$.

b) When $x = -2$,
$\dfrac{dy}{dx} = \dfrac{-16(-2)}{((-2)^2+4)^2} = \dfrac{32}{64} = \dfrac{1}{2}$, so the slope of the tangent line at $(-2,1)$ is $\dfrac{1}{2}$. Using the point-slope equation, we have:

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{1}{2}(x - (-2))$$
$$y - 1 = \frac{1}{2}x + 1$$
$$y = \frac{1}{2}x + 2$$

50. $y = \dfrac{\sqrt{x}}{x+1} = \dfrac{x^{1/2}}{x+1}$

$$\frac{dy}{dx} = \frac{(x+1)\cdot\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}\cdot(1)}{(x+1)^2}$$
$$= \frac{\frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2} - x^{1/2}}{(x+1)^2}$$
$$= \frac{-x^{1/2} + x^{-1/2}}{2(x+1)^2}$$

When $x = 1$, $y = \dfrac{\sqrt{1}}{(1+1)} = \dfrac{1}{2}$.

$$\frac{dy}{dx} = \frac{-(1)^{1/2} + (1)^{-1/2}}{2(1+1)^2} = 0$$

Therefore, the slope of the tangent line at $\left(1, \frac{1}{2}\right)$ is 0. The equation of the horizontal line passing through $\left(1, \frac{1}{2}\right)$ is $y = \frac{1}{2}$.

b) When $x = \dfrac{1}{4}$,

$$y = \frac{\sqrt{\frac{1}{4}}}{\left(\frac{1}{4}\right)+1} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{1}{2}\cdot\frac{4}{5} = \frac{2}{5}$$

$$\frac{dy}{dx} = \frac{-\left(\frac{1}{4}\right)^{1/2} + \left(\frac{1}{4}\right)^{-1/2}}{2\left(\frac{1}{4}+1\right)^2} = \frac{-\frac{1}{2}+2}{\frac{25}{8}} = \frac{3}{2}\cdot\frac{8}{25} = \frac{12}{25}$$

Therefore, the slope of the tangent line at $\left(\frac{1}{4}, \frac{2}{5}\right)$ is $\frac{12}{25}$. Using the point-slope equation, we have:

$$y - y_1 = m(x - x_1)$$
$$y - \frac{2}{5} = \frac{12}{25}\left(x - \frac{1}{4}\right)$$
$$y - \frac{2}{5} = \frac{12}{25}x - \frac{3}{25}$$
$$y = \frac{12}{25}x + \frac{7}{25}$$

51. $y = x^2 + \dfrac{3}{x-1}$

$$\frac{dy}{dx} = 2x + \frac{(x-1)(0) - 3(1)}{(x-1)^2}$$
$$= 2x - \frac{3}{(x-1)^2}$$

a) When $x = 2$, $y = (2)^2 + \dfrac{3}{2-1} = 4 + 3 = 7$,
and $\dfrac{dy}{dx} = 2(2) - \dfrac{3}{(2-1)^2} = 4 - 3 = 1$.
Therefore, the slope of the tangent line at $(2,7)$ is 1.
Using the point-slope equation, we have:

$$y - y_1 = m(x - x_1)$$
$$y - 7 = 1(x - 2)$$
$$y - 7 = x - 2$$
$$y = x + 5$$

b) When $x = 3$, $y = (3)^2 + \frac{3}{3-1} = 9 + \frac{3}{2} = \frac{21}{2}$,

and $\frac{dy}{dx} = 2(3) - \frac{3}{(3-1)^2} = 6 - \frac{3}{4} = \frac{21}{4}$.

Therefore, the slope of the tangent line at $\left(3, \frac{21}{2}\right)$ is $\frac{21}{4}$.

Using the point-slope equation, we have:

$$y - y_1 = m(x - x_1)$$
$$y - \frac{21}{2} = \frac{21}{4}(x - 3)$$
$$y - \frac{21}{2} = \frac{21}{4}x - \frac{63}{4}$$
$$y = \frac{21}{4}x - \frac{21}{4}$$

52. $y = \frac{4x}{1+x^2}$

$$\frac{dy}{dx} = \frac{(1+x^2)(4) - 4x(2x)}{(1+x^2)^2}$$
$$= \frac{4 + 4x^2 - 8x^2}{(1+x^2)^2}$$
$$= \frac{4 - 4x^2}{(1+x^2)^2}$$

a) When $x = 0$, $\frac{dy}{dx} = \frac{4 - 4(0)^2}{\left(1 + (0)^2\right)^2} = 4$.

Therefore, the slope of the tangent line at $(0,0)$ is 4.

Using the point-slope equation, we have:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 4(x - 0)$$
$$y = 4x$$

b) When $x = -1$, $\frac{dy}{dx} = \frac{4 - 4(-1)^2}{\left(1 + (-1)^2\right)^2} = \frac{0}{4} = 0$.

Therefore, the slope of the tangent line at $(-1,-2)$ is 0. The equation of the horizontal line passing through $(-1,-2)$ is $y = -2$.

53. The average cost of producing x items is $A_C(x) = \frac{C(x)}{x}$. Therefore,

$$A_C(x) = \frac{750 + 34x - 0.068x^2}{x}.$$

Next, we take the derivative using the Quotient Rule to find the rate at which average cost is changing.

$$A_C'(x) = \frac{d}{dx}\left(\frac{750 + 34x - 0.068x^2}{x}\right)$$
$$A_C'(x) = \frac{x(34 - 0.136x) - (750 + 34x - 0.068x^2)(1)}{(x)^2}$$
$$= \frac{34x - 0.136x^2 - (750 + 34x - 0.068x^2)}{x^2}$$
$$= \frac{-0.068x^2 - 750}{x^2}$$

Substituting 175 for *x,* we have

$$A_C'(175) == \frac{-0.068(175)^2 - 750}{(175)^2}$$
$$= \frac{-2082.5 - 750}{30,625}$$
$$\approx -0.09249$$

Therefore, when 175 belts have been produced, average cost is changing at a rate of -0.09249 dollars per belt.

54. $A_C(x) = \frac{C(x)}{x}$

$$A_C(x) = \frac{375 + 0.75x^{3/4}}{x}$$
$$A_C'(x) = \frac{x\left(0.75\left(\frac{3}{4}x^{-1/4}\right)\right) - \left(375 + 0.75x^{3/4}\right)(1)}{x^2}$$
$$= \frac{\frac{9}{16}x^{3/4} - 375 - \frac{3}{4}x^{3/4}}{x^2}$$
$$= \frac{-\frac{3}{16}x^{3/4} - 375}{x^2}$$

The solution is continued on the next page.

Substituting 81 for x into $A_c\text{'}(x)$ from the previous page, we have:

$$A_C\text{'}(81)=\frac{\frac{-3}{16}(81)^{3/4}-375}{(81)^2}$$
$$=-0.05792753$$
$$\approx -0.0579$$

Therefore, when 81 bottles of barbecue sauce have been produced, the average cost is changing at a rate of -0.0579 dollars per bottle.

55. The average revenue of producing x items is $A_R(x)=\frac{R(x)}{x}$. Therefore,

$$A_R(x)=\frac{45x^{9/10}}{x}=\frac{45}{x^{1/10}}.$$

Next, we take the derivative using the Quotient Rule to find the rate at which average revenue is changing.

$$A_R\text{'}(x)=\frac{d}{dx}\left(\frac{45}{x^{1/10}}\right)$$
$$=\frac{x^{1/10}(0)-(45)\left(\frac{1}{10}x^{-9/10}\right)}{\left(x^{1/10}\right)^2}$$
$$=\frac{-\frac{45}{10}x^{-9/10}}{x^{2/10}}$$
$$=-\frac{9}{2x^{11/10}}$$

Substituting 175 for x, we have

$$A_R\text{'}(175)=-\frac{9}{2(175)^{11/10}}$$
$$=-\frac{9}{586.64023}$$
$$=-0.0153$$

Therefore, when 175 belts have been produced, average revenue is changing at a rate of -0.0153 dollars per belt.

56. $A_R(x)=\frac{R(x)}{x}$

$$A_R(x)=\frac{7.5x^{0.7}}{x}=\frac{7.5}{x^{0.3}}$$

$$A_R\text{'}(x)=\frac{x^{0.3}(0)-(7.5)\left(0.3x^{-0.7}\right)}{\left(x^{0.3}\right)^2}$$
$$=\frac{-2.25x^{-0.7}}{x^{0.6}}$$
$$=-\frac{2.25}{x^{1.3}}$$

Substituting 81 for x, we have:

$$A_R\text{'}(81)=-\frac{2.25}{(81)^{1.3}}$$
$$=-0.007432792$$
$$\approx -0.0074$$

Therefore, when 81 bottles of barbecue sauce have been produce, the average revenue is changing at rate of -0.0074 dollars per bottle.

57. $A_P(x)=\frac{P(x)}{x}=\frac{R(x)-C(x)}{x}$

From Exercises 53 and 55, we know that

$$A_P(x)=\frac{45x^{9/10}-\left(750+34x-0.068x^2\right)}{x}$$
$$=\frac{0.068x^2-34x-750+45x^{9/10}}{x}$$

Using the Quotient Rule to take the derivative, we have:

$$A_P\text{'}(x)=\frac{x\left(0.136x-34+40.5x^{-1/10}\right)}{(x)^2}-\frac{\left(0.068x^2-34x-750+45x^{9/10}\right)(1)}{(x)^2}$$
$$=\frac{0.068x^2+750-4.5x^{9/10}}{x^2}$$

Substituting 175 for x, we have:

$$A_P\text{'}(175)=\frac{0.068(175)^2+750-4.5(175)^{9/10}}{(175)^2}$$
$$=\frac{2082.5+750-469.8365}{30,625}$$
$$=\frac{2362.6635}{30,625}$$
$$\approx 0.0772$$

When 175 belts have been produced and sold, the average profit is changing at a rate of 0.0772 dollars per belt.

Alternatively, we could have used the information in Exercises 53 and 55 to find the rate of change of average profit when 175 belts are produced and sold. Notice that

$$\begin{aligned} A_P'(x) &= A_R'(x) - A_C'(x) \\ &= -0.0153 - (-0.0925) \\ &= 0.0772 \end{aligned}$$

58. $$\begin{aligned} A_P'(x) &= A_R'(x) - A_C'(x) \\ &= -0.0074 - (-0.0579) \\ &= 0.0505 \end{aligned}$$

Therefore, when 81 bottles are produced and sold, average profit is changing at rate of 0.050 dollars per bottle.

59. The average profit of producing *x* items is $A_P(x) = \dfrac{R(x) - C(x)}{x}$. Therefore,

$$\begin{aligned} A_P(x) &= \frac{65x^{0.9} - \left(4300 + 2.1x^{0.6}\right)}{x} \\ &= \frac{65x^{0.9} - 2.1x^{0.6} - 4300}{x}. \end{aligned}$$

Using the Quotient Rule to take the derivative, we have

$$\begin{aligned} &A_P'(x) \\ &= \frac{x\left(65\left(0.9x^{-0.1}\right) - 2.1\left(0.6x^{-0.4}\right)\right) - \left(65x^{0.9} - 2.1x^{0.6} - 4300\right)(1)}{(x)^2} \\ &= \frac{58.5x^{0.9} - 1.26x^{0.6} - \left(65x^{0.9} - 2.1x^{0.6} - 4300\right)}{x^2} \\ &= \frac{-6.5x^{0.9} + 0.84x^{0.6} + 4300}{x^2} \end{aligned}$$

Substituting 50 for *x,* we have

$$\begin{aligned} A_P'(50) &= \frac{-6.5(50)^{0.9} + 0.84(50)^{0.6} + 4300}{(50)^2} \\ &= \frac{4089.00428745}{2500} \\ &= 1.63560171 \\ &\approx 1.64 \end{aligned}$$

Therefore, when 50 vases have been produced and sold, the average profit is changing at rate of 1.64 dollars per vase.

60. $A_P(x) = \dfrac{R(x) - C(x)}{x}$.

Therefore,

$$\begin{aligned} A_P(x) &= \frac{75x^{0.8} - \left(900 + 18x^{0.7}\right)}{x} \\ &= \frac{75x^{0.8} - 18x^{0.7} - 900}{x}. \end{aligned}$$

$$\begin{aligned} &A_P'(x) \\ &= \frac{x\left(60x^{-0.2} - 12.6x^{-0.3}\right) - \left(75x^{0.8} - 18x^{0.7} - 900\right)(1)}{(x)^2} \\ &= \frac{-15x^{0.8} + 5.4x^{0.7} + 900}{x^2} \end{aligned}$$

Substituting 20 for *x,* we have

$$\begin{aligned} A_P'(20) &= \frac{-15(20)^{0.8} + 5.4(20)^{0.7} + 900}{(20)^2} \\ &= \frac{779.18169591}{400} \\ &= 1.94795424 \\ &\approx 1.95 \end{aligned}$$

Therefore, when 20 skateboards have been produced and sold, the average profit is changing at rate of 1.95 dollars per skateboard.

61. $P(t) = 567 + t\left(36t^{0.6} - 104\right)$

a) Using the Product Rule and remembering that the derivative of a constant is 0, we have

$$\begin{aligned} P'(t) &= 0 + t\left(36\left(0.6t^{-0.4}\right)\right) + \left(36t^{0.6} - 104\right)(1) \\ &= 21.6t^{0.6} + 36t^{0.6} - 104 \\ &= 57.6t^{0.6} - 104 \end{aligned}$$

b) Substituting 45 for *t*, we have:

$$\begin{aligned} P'(45) &= 57.6(45)^{0.6} - 104 \\ &= 565.39243291 - 104 \\ &= 461.39243291 \end{aligned}$$

c) $P'(45)$ is the rate of change of the gross domestic product 45 years after 1960, or in 2005. In other words, the U.S. Gross domestic product was increasing at a rate of 461.39243291 billion dollars per year in 2005.

62. $P(t)=\dfrac{500t}{2t^2+9}$

a) $P'(t)=\dfrac{(2t^2+9)(500)-(500t)(4t)}{(2t^2+9)^2}$

$=\dfrac{1000t^2+4500-2000t^2}{(2t^2+9)^2}$

$=\dfrac{-1000t^2+4500}{(2t^2+9)^2}$

$=\dfrac{-500(2t^2-9)}{(2t^2+9)^2}$

b) $P(12)=\dfrac{500(12)}{2(12)^2+9}=\dfrac{6000}{297}=20.2020$

After 12 years, the city's population is approximately 20.2020 thousand people or 20,202 people.

c) $P'(12)=\dfrac{-500(2(12)^2-9)}{(2(12)^2+9)^2}$

$P'(12)=\dfrac{-139,500}{88,209}$

$=-1.58147128$

The city population is growing at rate of -1.581 thousand people per year. In other words, the city population is declining at a rate of 1,581 people per year.

63. $T(t)=\dfrac{4t}{t^2+1}+98.6$

a) $T'(t)=\dfrac{(t^2+1)(4)-(4t)(2t)}{(t^2+1)^2}+0$

$=\dfrac{4t^2+4-8t^2}{(t^2+1)^2}$

$=\dfrac{-4t^2+4}{(t^2+1)^2}$

$=\dfrac{-4(t^2-1)}{(t^2+1)^2}$

b) $T(2)=\dfrac{4(2)}{(2)^2+1}+98.6=\dfrac{8}{5}+98.6=100.2$

After 2 hours, Gloria's temperature is approximately 100.2 degrees Fahrenheit.

c) $T'(2)=\dfrac{-4((2)^2-1)}{((2)^2+1)^2}=\dfrac{-12}{25}=-0.48$

After 2 hours, Gloria's temperature is changing at rate of -0.48 degrees per hour.

64. $f(x)=\dfrac{7-\frac{3}{2x}}{\frac{4}{x^2}+5}$

Simplifying the function we have:

$f(x)=\dfrac{7-\frac{3}{2x}}{\frac{4}{x^2}+5}\cdot\dfrac{2x^2}{2x^2}$

$=\dfrac{7(2x^2)-3(x)}{4(2)+5(2x^2)}$

$=\dfrac{14x^2-3x}{8+10x^2}$

We apply the Quotient Rule to take the derivative.

$f'(x)=\dfrac{(8+10x^2)(28x-3)-(14x^2-3x)(20x)}{(8+10x^2)^2}$

$=\dfrac{280x^3-30x^2+224x-24-280x^3+60x^2}{(8+10x^2)^2}$

$=\dfrac{30x^2+224x-24}{(8+10x^2)^2}$

The previous derivative can be simplified as follows:

$f'(x)=\dfrac{15x^2+112x-12}{2(5x^2+4)^2};\quad x\neq 0$.

65. $y(t)=5t(t-1)(2t+3)$

First, group the factors of $y(t)$ in order to apply the product rule.

$y(t)=[5t(t-1)]\cdot(2t+3)$

Now we calculate the derivative at the top of the next page.

Notice that when we take the derivative of the first term, $\left[5t(t-1)\right]$ we will have to apply the Product Rule again.

$$y'(t)=\left[5t(t-1)\right](2)+(2t+3)\left[\underbrace{(5t)(1)+(t-1)(5)}_{\text{Product Rule for }\left[5t(t-1)\right]}\right]$$
$$=10t(t-1)+(2t+3)[5t+5t-5]$$
$$=10t(t-1)+(2t+3)(10t-5)$$

The previous derivative can be simplified as follows:

$$y'(t)=10t^2-10t+20t^2+30t-10t-15$$
$$=30t^2+10t-15$$

66. $f(x)=\left[x\left(3x^3+6x-2\right)\right]\left(3x^4+7\right)$

$$f'(x)=\left[x\left(3x^3+6x-2\right)\right]\left(12x^3\right)+\left(3x^4+7\right)\left[x\left(9x^2+6\right)+\left(3x^3+6x-2\right)(1)\right]$$
$$=36x^7+72x^5-24x^4+\left(3x^4+7\right)\left[9x^3+6x+3x^3+6x-2\right]$$
$$=36x^7+72x^5-24x^4+\left(3x^4+7\right)\left[12x^3+12x-2\right]$$
$$=36x^7+72x^5-24x^4+36x^7+36x^5-6x^4+84x^3+84x-14$$
$$=72x^7+108x^5-30x^4+84x^3+84x-14$$

67. $g(x)=\left(x^3-8\right)\cdot\frac{x^2+1}{x^2-1}$

We will begin by applying the Product Rule.

$$g'(x)=\left(x^3-8\right)\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)+\frac{x^2+1}{x^2-1}\cdot\frac{d}{dx}\left(x^3-8\right)$$

Notice, that we will have to apply the Quotient Rule to take the derivative of $\frac{x^2+1}{x^2-1}$.

$$g'(x)=\left(x^3-8\right)\frac{\left(x^2-1\right)(2x)-\left(x^2+1\right)(2x)}{\left(x^2-1\right)^2}+\left(\frac{x^2+1}{x^2-1}\right)\cdot\left(3x^2\right)$$
$$=\left(x^3-8\right)\frac{-4x}{\left(x^2-1\right)^2}+\left(\frac{x^2+1}{x^2-1}\right)\cdot\left(3x^2\right)$$

The derivative on the previous column can be simplified as follows.

$$g'(x)=\frac{-4x\left(x^3-8\right)}{\left(x^2-1\right)^2}+\frac{3x^2\left(x^2+1\right)}{x^2-1}$$
$$=\frac{-4x^4+32x}{\left(x^2-1\right)^2}+\frac{3x^4+3x^2}{x^2-1}\cdot\frac{x^2-1}{x^2-1}$$
$$=\frac{-4x^4+32x+3x^6-3x^2}{\left(x^2-1\right)^2}$$
$$=\frac{3x^6-4x^4-3x^2+32x}{\left(x^2-1\right)^2}$$

68. $f(t)=\left(t^5+3\right)\cdot\frac{t^3-1}{t^3+1}$

$$f'(t)=\left(t^5+3\right)\left[\frac{\left(t^3+1\right)\left(3t^2\right)-\left(t^3-1\right)\left(3t^2\right)}{\left(t^3+1\right)^2}\right]+\left(\frac{t^3-1}{t^3+1}\right)\left(5t^4\right)$$
$$=\left(t^5+3\right)\left[\frac{6t^2}{\left(t^3+1\right)^2}\right]+\left(\frac{t^3-1}{t^3+1}\right)5t^4$$
$$=\frac{6t^2\left(t^5+3\right)}{\left(t^3+1\right)^2}+\frac{5t^4\left(t^3-1\right)}{t^3+1}$$

69. $f(x)=\frac{(x-1)\left(x^2+x+1\right)}{x^4-3x^3-5}$

First we will group the numerator, to apply the Quotient Rule. Remember that we will have to apply the Product Rule when taking the derivative of the numerator.

$$f(x)=\frac{\left[(x-1)\left(x^2+x+1\right)\right]}{x^4-3x^3-5}$$

We calculate the derivative at the top of the next page.

Calculating the derivative from the function on the previous page, we have:

$$f'(x)$$
$$=\frac{\left(x^4-3x^3-5\right)\left[(x-1)(2x+1)+\left(x^2+x+1\right)(1)\right]}{\left(x^4-3x^3-5\right)^2}-\frac{\left[(x-1)\left(x^2+x+1\right)\right]\left(4x^3-9x^2\right)}{\left(x^4-3x^3-5\right)^2}$$
$$=\frac{\left(x^4-3x^3-5\right)\left[2x^2-x-1+x^2+x+1\right]}{\left(x^4-3x^3-5\right)^2}-\frac{\left[x^3-1\right]\left(4x^3-9x^2\right)}{\left(x^4-3x^3-5\right)^2}$$
$$=\frac{\left(x^4-3x^3-5\right)\left[3x^2\right]-\left[x^3-1\right]\left(4x^3-9x^2\right)}{\left(x^4-3x^3-5\right)^2}$$
$$=\frac{3x^6-9x^5-15x^2-4x^6+9x^5+4x^3-9x^2}{\left(x^4-3x^3-5\right)^2}$$
$$=\frac{-x^6+4x^3-24x^2}{\left(x^4-3x^3-5\right)^2}$$

70. $f(x)=\frac{x}{x+1}$ and $g(x)=\frac{-1}{x+1}$

a) $f'(x)=\frac{(x+1)(1)-x(1)}{(x+1)^2}=\frac{1}{(x+1)^2}$

b) $g'(x)=\frac{(x+1)(0)-(-1)(1)}{(x+1)^2}=\frac{1}{(x+1)^2}$

c) The two functions have the same rate of change.

71. $f(x)=\frac{x^2}{x^2-1}$ and $g(x)=\frac{1}{x^2-1}$

a) $f'(x)=\frac{\left(x^2-1\right)(2x)-x^2(2x)}{\left(x^2-1\right)^2}=\frac{-2x}{\left(x^2-1\right)^2}$

b) $g'(x)=\frac{\left(x^2-1\right)(0)-1(2x)}{\left(x^2-1\right)^2}=\frac{-2x}{\left(x^2-1\right)^2}$

c) The two functions have the same rate of change, therefore the graphs will have the same slope.

72. Suppose $F(x)=f(x)\cdot g(x)\cdot h(x)$, where $f(x)$ is the "first" function, $g(x)$ is the "second" function and $h(x)$ is the third function. Using the associative law of multiplication, we can write $F(x)=\left[f(x)\cdot g(x)\right]\cdot h(x)$. Then using the Product Rule we have:

$$\begin{aligned}F'(x)&=\frac{d}{dx}\left(\left[f(x)\cdot g(x)\right]\cdot h(x)\right)\\&=\left[f(x)g(x)\right]h'(x)+h(x)\left[f(x)g(x)\right]'\\&=\left[f(x)g(x)\right]h'(x)+\\&\quad h(x)\left[f(x)g'(x)+g(x)f'(x)\right]\\&=f(x)g(x)h'(x)+f(x)g'(x)h(x)+\\&\quad f'(x)g(x)h(x)\end{aligned}$$

The derivative of the product of three functions is the first function times the second function times the derivative of the third functions plus the first function times the derivative of the second function times the third function plus the derivative of the first function times the second function times the third function.

73. In general the derivative of the reciprocal of a function is not the reciprocal of the derivative. Let $f'(x)$ be the derivative of $f(x)$.

Therefore, the reciprocal of the derivative is $\frac{1}{f'(x)}$.

The reciprocal of the function is $\frac{1}{f(x)}$, using the Quotient Rule, we find the derivative of the reciprocal.

$$\frac{d}{dx}\left(\frac{1}{f(x)}\right)=\frac{f(x)(0)-(1)f'(x)}{\left(f(x)\right)^2}=-\frac{f'(x)}{\left(f(x)\right)^2}$$

Clearly, the derivative of the reciprocal is not equal to the reciprocal of the derivative.

74. $R(Q)=Q^2\left(\frac{k}{2}-\frac{Q}{3}\right)$

a) $$\begin{aligned}\frac{dR}{dQ}&=Q^2\left(-\frac{1}{3}\right)+\left(\frac{k}{2}-\frac{Q}{3}\right)(2Q)\\&=-\frac{1}{3}Q^2+\frac{k}{2}2Q-\frac{2Q^2}{3}\\&=-Q^2+kQ\end{aligned}$$

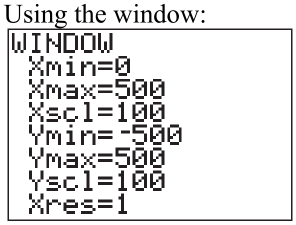
b) The derivative found in part (a) tells us the rate of change of the reaction with respect to a small change in the quantity of the dose. If the reaction is measured in blood pressure, then the derivative tells us at a dosage Q, how much change in millimeters of mercury is seen due to a small change in the dosage. If the reaction is measured in temperature, the derivative tells us at a dosage Q, how much change in degrees Fahrenheit is seen due to a small change in the dosage.

75. a) Definition of the derivative.
b) Adding and subtracting the same quantity is the same as adding 0.
c) The limit of a sum is the sum of the limits.
d) Factoring common factors.
e) The limit of a product is the product of the limits and $\lim_{h\to 0} f(x+h) = f(x)$.
f) Definition of the derivative.
g) Using Leibniz's notation.

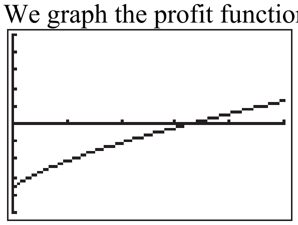
76. The break-even point occurs when $P(x)=0$.

$$P(x) = R(x) - C(x)$$
$$= 7.5x^{0.7} - 375 - 0.75x^{3/4}$$

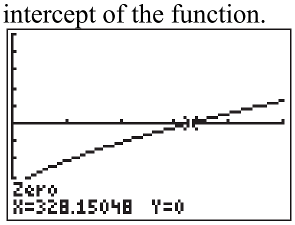
Using the window:



We graph the profit function on the calculator.



The break-even point will be the zero or x-intercept of the function.



We see that the break-even point occurs at $x = 328$ bottles.

The solution is continued on the next column.

The profit is changing at rate of

$$P'(x) = 5.25x^{-0.3} - 0.5625x^{-1/4}$$

Substituting 328 for x we have:

$$P'(328) = 5.25(328)^{-0.3} - 0.5625(328)^{-1/4}$$
$$= 0.791239$$
$$\approx 0.79$$

Therefore, at the break-even point, profit is increasing at a rate of 0.79 dollars per bottle, or 79 cents per bottle.

From Exercises 54, 56, and 58 we know that:

$$A_P'(x) = A_R'(x) - A_C'(x)$$
$$= -\frac{2.25}{x^{1.3}} - \frac{-\frac{3}{16}x^{3/4} - 375}{x^2}$$
$$= \frac{3x^{3/4} - 36x^{0.7} + 6000}{16x^2}$$

Substituting 328 for x we get:

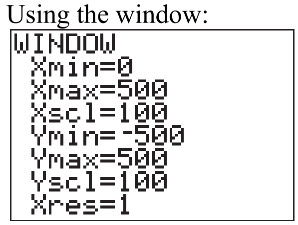
$$A_P'(328) = \frac{3(328)^{3/4} - 36(328)^{0.7} + 6000}{16(328)^2}$$
$$\approx 0.00241342$$
$$\approx 0.0024$$

At the break-even point, average profit is changing at a rate of 0.0024 dollars per bottle.

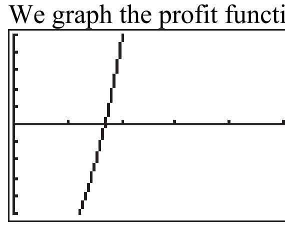
77. The break-even point occurs when $P(x)=0$.

$$P(x) = R(x) - C(x)$$
$$= 45x^{9/10} - \left(750 + 34x - 0.068x^2\right)$$
$$= 0.068x^2 - 34x - 750 + 45x^{9/10}$$

Using the window:

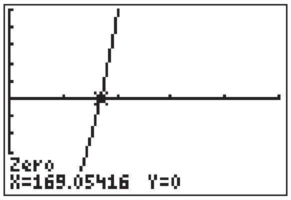


We graph the profit function on the calculator.



The solution is continued on the next page.

The break-even point will be the zero of the function. Using the zero finder on the calculator we have:



We see that the break-even point occurs at $x = 170$ belts.

The profit is changing at rate of

$$P(x) = 0.068x^2 - 34x - 750 + 45x^{9/10}$$

$$P'(x) = 0.136x - 34 + 40.5x^{-1/10}$$

Substituting 170 for x we have:

$$P'(170) = 0.136(170) - 34 + 40.5(170)^{-1/10}$$
$$= 13.353$$
$$\approx 13.35$$

Therefore, at the break-even point, profit is increasing at a rate of 13.35 dollars per belt.

From Exercise 57 we know that:

$$A_P'(x) = \frac{0.068x^2 + 750 - 4.5x^{9/10}}{x^2}$$

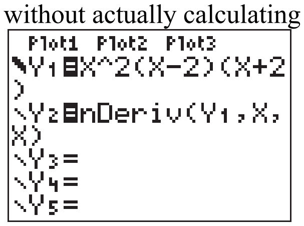
Substituting 170 for x we get:

$$A_P'(170) = \frac{0.068(170)^2 + 750 - 4.5(170)^{9/10}}{(170)^2}$$
$$\approx 0.07811 \approx 0.078$$

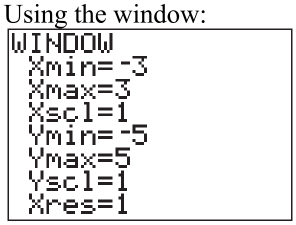
At the break-even point, average profit is changing at a rate of 0.078 dollars per belt.

78. $f(x) = x^2(x-2)(x+2)$

Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

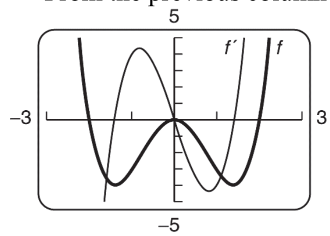


Using the window:



The graph is displayed on the next column.

From the previous column we have:

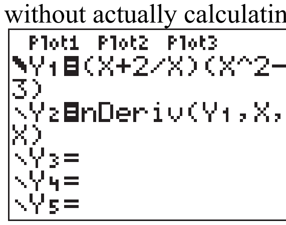


Note, the function $f(x)$ is the thicker graph.

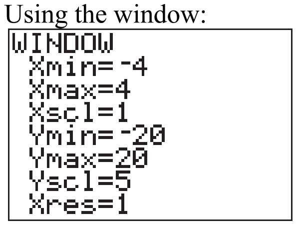
The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency. We estimate the points at which the tangent lines are horizontal are $(0,0), (-1.414,-4)$, and $(1.414,-4)$.

79. $f(x) = \left(x + \frac{2}{x}\right)(x^2 - 3)$

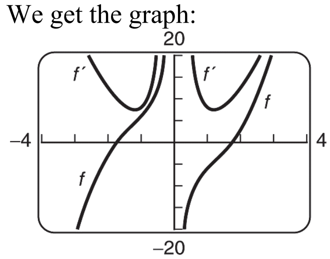
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:

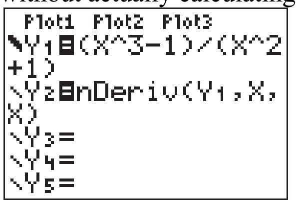


Note, the function $f(x)$ is the thicker graph.

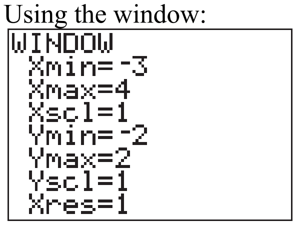
The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. We can see that the derivative never intersects the x-axis, therefore, there are no points at which the tangent line is horizontal.

80. $f\left(x\right)=\dfrac{x^3-1}{x^2+1}$

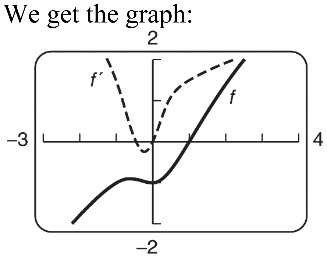
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:



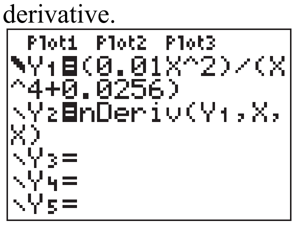
Note, the function $f\left(x\right)$ is the solid graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative.

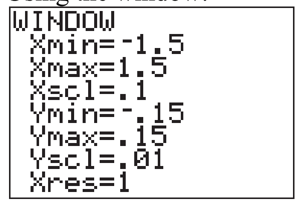
Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency. We estimate the points at which the tangent lines are horizontal are $\left(-0.596,-0.894\right)$ and $\left(0,-1\right)$.

81. $f\left(x\right)=\dfrac{0.01x^2}{x^4+0.0256}$

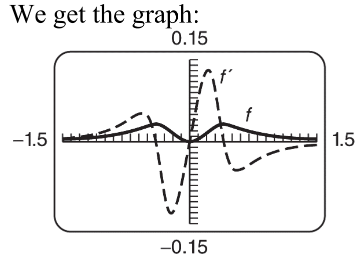
Using the calculator, we graph the function and the derivative in the same window.
We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:



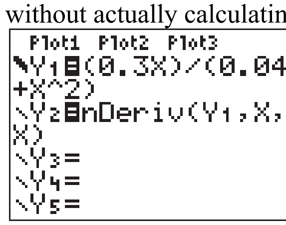
Note, the function $f\left(x\right)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

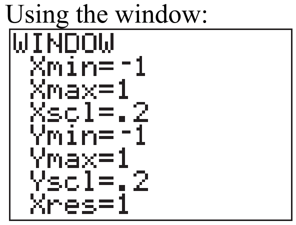
We estimate the points at which the tangent lines are horizontal are
$\left(-0.4,0.03125\right)$, $\left(0,0\right)$, and $\left(0.4,0.03125\right)$.

82. $f\left(x\right)=\dfrac{0.3x}{0.04+x^2}$

Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

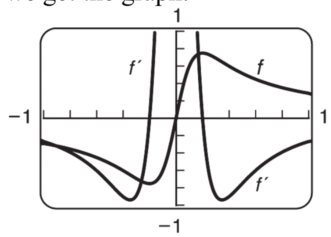


Using the window:



The solution is continued on the next page.

Using the information from the previous page, we get the graph:



Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

We estimate the points at which the tangent lines are horizontal are $(-0.2,-0.75)$ and $(0.2,0.75)$.

83. $f(x)=\dfrac{4x}{x^2+1}$

Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

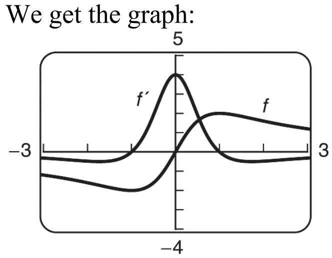
```
Plot1 Plot2 Plot3
\Y1=(4X)/(X^2+1)

\Y2=nDeriv(Y1,X,
X)
\Y3=
\Y4=
\Y5=
```

Using the window:

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-4
 Ymax=5
 Yscl=1
 Xres=1
```

We get the graph:

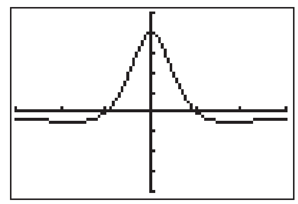


Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

We estimate the points at which the tangent lines are horizontal are $(-1,-2)$ and $(1,2)$.

84. The graph of $y_3=\dfrac{4-4x^2}{\left(x^2+1\right)^2}$ is:



This graph appears to be the correct derivative of the function in Exercise 83. Using the Quotient Rule we can verify this result.

$$f(x)=\frac{4x}{x^2+1}$$

$$f'(x)=\frac{4-4x^2}{\left(x^2+1\right)^2}$$

Exercise Set 1.7

1. $y=(3-2x)^2$

Using the Extended Power Rule:

$$\frac{dy}{dx}=\frac{d}{dx}\left[(3-2x)^2\right]$$
$$=2(3-2x)^{2-1}\cdot\frac{d}{dx}(3-2x)$$
$$=2(3-2x)(-2)$$
$$=8x-12$$

Simplifying the function first, we have:

$$y=(3-2x)^2$$
$$=(3-2x)(3-2x)$$
$$=4x^2-12x+9$$

We take the derivative using the Power Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(4x^2-12x+9\right)$$
$$=\frac{d}{dx}\left(4x^2\right)-\frac{d}{dx}(12x)+\frac{d}{dx}(9)$$
$$=8x-12$$

The results are the same.

2. $y=(2x+1)^2$

Using the Extended Power Rule:

$$\frac{dy}{dx}=\frac{d}{dx}\left[(2x+1)^2\right]$$
$$=2(2x+1)^{2-1}(2)$$
$$=8x+4$$

Simplifying the function first, we have:

$$y=(2x+1)^2=4x^2+4x+1$$

Now we take the derivative using the Power Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(4x^2+4x+1\right)$$
$$=8x+4$$

The results are the same.

3. $y=(7-x)^{55}$

Using the Extended Power Rule:

$$\frac{dy}{dx}=\frac{d}{dx}\left[(7-x)^{55}\right]$$
$$=55(7-x)^{55-1}\cdot\frac{d}{dx}(7-x)$$

$$\frac{dy}{dx}=55(7-x)^{54}(-1)$$
$$=-55(7-x)^{54}$$

4. $y=(8-x)^{100}$

$$\frac{dy}{dx}=100(8-x)^{99}(-1)$$
$$=-100(8-x)^{99}$$

5. $y=\sqrt{1-x}=(1-x)^{1/2}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}(1-x)^{1/2}$$
$$=\frac{1}{2}(1-x)^{1/2-1}\cdot\frac{d}{dx}(1-x)$$
$$=\frac{1}{2}(1-x)^{-1/2}\cdot(-1)$$
$$=\frac{-1}{2(1-x)^{1/2}}$$
$$=\frac{-1}{2\sqrt{1-x}}$$

6. $y=\sqrt{1+8x}=(1+8x)^{1/2}$

$$\frac{dy}{dx}=\frac{d}{dx}\left[(1+8x)^{1/2}\right]$$
$$=\frac{1}{2}(1+8x)^{-1/2}\cdot(8)$$
$$=\frac{8}{2(1+8x)^{1/2}}$$
$$=\frac{4}{\sqrt{1+8x}}$$

7. $y=\sqrt{3x^2-4}=\left(3x^2-4\right)^{1/2}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(3x^2-4\right)^{1/2}\right]$$
$$=\frac{1}{2}\left(3x^2-4\right)^{-1/2}\cdot\frac{d}{dx}\left(3x^2-4\right)$$
$$=\frac{1}{2\left(3x^2-4\right)^{1/2}}\cdot(6x)$$
$$=\frac{3x}{\sqrt{3x^2-4}}$$

8. $y=\sqrt{4x^2+1}=\left(4x^2+1\right)^{\frac{1}{2}}$

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(4x^2+1\right)^{\frac{1}{2}}\right]$$

$$\frac{dy}{dx}=\frac{1}{2}\left(4x^2+1\right)^{-\frac{1}{2}}(8x)$$

$$=\frac{8x}{2\left(4x^2+1\right)^{\frac{1}{2}}}$$

$$=\frac{4x}{\sqrt{4x^2+1}}$$

9. $y=\left(4x^2+1\right)^{-50}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(4x^2+1\right)^{-50}\right]$$

$$=-50\left(4x^2+1\right)^{-50-1}\cdot\frac{d}{dx}\left(4x^2+1\right)$$

$$=-50\left(4x^2+1\right)^{-51}\cdot(8x)$$

$$=-400x\left(4x^2+1\right)^{-51}$$

$$=\frac{-400x}{\left(4x^2+1\right)^{51}}$$

10. $y=\left(8x^2-6\right)^{-40}$

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(8x^2-6\right)^{-40}\right]$$

$$=-40\left(8x^2-6\right)^{-41}\cdot(16x)$$

$$=-640x\left(8x^2-6\right)^{-41}$$

$$=\frac{-640x}{\left(8x^2-6\right)^{41}}$$

11. $y=(x-4)^8(2x+3)^6$

Using the Product Rule, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[(x-4)^8(2x+3)^6\right]$$

$$=(x-4)^8\frac{d}{dx}(2x+3)^6+(2x+3)^6\frac{d}{dx}(x-4)^8$$

Next, we will apply the Extended Power Rule.

$$\frac{dy}{dx}=(x-4)^8\left[6(2x+3)^{6-1}\cdot\frac{d}{dx}(2x+3)\right]+$$
$$(2x+3)^6\left[8(x-4)^{8-1}\cdot\frac{d}{dx}(x-4)\right]$$

$$=(x-4)^8\left[6(2x+3)^5(2)\right]+$$
$$(2x+3)^6\left[8(x-4)^7(1)\right]$$

$$=12(x-4)^8(2x+3)^5+8(2x+3)^6(x-4)^7$$

Factoring out common factors, we have:

$$\frac{dy}{dx}=4(x-4)^7(2x+3)^5\left[3(x-4)+2(2x+3)\right]$$

$$=4(x-4)^7(2x+3)^5\left[3x-12+4x+6\right]$$

$$=4(x-4)^7(2x+3)^5(7x-6)$$

12. $y=(x+5)^7(4x-1)^{10}$

$$\frac{dy}{dx}=(x+5)^7\left[10(4x-1)^9(4)\right]+$$
$$(4x-1)^{10}\left[7(x+5)^6(1)\right]$$

$$=40(x+5)^7(4x-1)^9+7(4x-1)^{10}(x+5)^6$$

$$=(x+5)^6(4x-1)^9\left[40(x+5)+7(4x-1)\right]$$

$$=(x+5)^6(4x-1)^9\left[40x+200+28x-7\right]$$

$$=(x+5)^6(4x-1)^9(68x+193)$$

13. $y=\frac{1}{(4x+5)^2}=(4x+5)^{-2}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[(4x+5)^{-2}\right]$$

$$=-2(4x+5)^{-2-1}\cdot\frac{d}{dx}(4x+5)$$

$$=-2(4x+5)^{-3}\cdot(4)$$

$$=-8(4x+5)^{-3}$$

$$=\frac{-8}{(4x+5)^3}$$

14. $y=\dfrac{1}{(3x+8)^2}=(3x+8)^{-2}$

$$\frac{dy}{dx}=\frac{d}{dx}\left[(3x+8)^{-2}\right]$$
$$=-2(3x+8)^{-3}\cdot(3)$$
$$=\frac{-6}{(3x+8)^3}$$

15. $y=\dfrac{4x^2}{(7-5x)^3}$

First, we use the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{4x^2}{(7-5x)^3}\right]$$
$$=\frac{(7-5x)^3\frac{d}{dx}\left(4x^2\right)-4x^2\frac{d}{dx}(7-5x)^3}{\left((7-5x)^3\right)^2}$$

Next, using the Extended Power Rule, we have:

$$\frac{dy}{dx}=\frac{(7-5x)^3(8x)-4x^2\left[3(7-5x)^2(-5)\right]}{(7-5x)^6}$$
$$=\frac{8x(7-5x)^3+60x^2(7-5x)^2}{(7-5x)^6}$$
$$=\frac{(7-5x)^2\left[8x(7-5x)+60x^2\right]}{(7-5x)^6} \quad \text{Factoring}$$
$$=\frac{56x-40x^2+60x^2}{(7-5x)^4} \quad \text{Dividing common factors}$$
$$=\frac{20x^2+56x}{(7-5x)^4}$$
$$=\frac{4x(5x+14)}{(7-5x)^4}$$

16. $y=\dfrac{7x^3}{(4-9x)^5}$

$$\frac{dy}{dx}=\frac{(4-9x)^5\left(21x^2\right)-7x^3\left[5(4-9x)^4(-9)\right]}{\left((4-9x)^5\right)^2}$$
$$=\frac{21x^2(4-9x)^5+315x^3(4-9x)^4}{(4-9x)^{10}}$$
$$=\frac{21x^2(4-9x)^4\left[(4-9x)+15x\right]}{(4-9x)^{10}}$$
$$\frac{dy}{dx}=\frac{21x^2(4+6x)}{(4-9x)^6}$$
$$=\frac{42x^2(3x+2)}{(4-9x)^6}$$

17. $f(x)=\left(3+x^3\right)^5-\left(1+x^7\right)^4$

Using the Difference Rule and then the Extended Power Rule we have:

$$f'(x)=\frac{d}{dx}\left[\left(3+x^3\right)^5-\left(1+x^7\right)^4\right]$$
$$=\frac{d}{dx}\left(3+x^3\right)^5-\frac{d}{dx}\left(1+x^7\right)^4$$
$$=5\left(3+x^3\right)^{5-1}\left(\frac{d}{dx}\left(3+x^3\right)\right)-4\left(1+x^7\right)^{4-1}\left(\frac{d}{dx}\left(1+x^7\right)\right)$$
$$=5\left(3+x^3\right)^4\left(3x^2\right)-4\left(1+x^7\right)^3\left(7x^6\right)$$
$$=15x^2\left(3+x^3\right)^4-28x^6\left(1+x^7\right)^3$$

18. $f(x)=\left(1+x^3\right)^3-\left(2+x^8\right)^4$

$$f'(x)=\frac{d}{dx}\left[\left(1+x^3\right)^3-\left(2+x^8\right)^4\right]$$
$$=3\left(1+x^3\right)^2\left(3x^2\right)-4\left(2+x^8\right)^3\left(8x^7\right)$$
$$=9x^2\left(1+x^3\right)^2-32x^7\left(2+x^8\right)^3$$

19. $f(x)=x^2+(200-x)^2$

Using the Sum Rule and the Extended Power Rule, we have:

$$f'(x)=\frac{d}{dx}\left[x^2+(200-x)^2\right]$$
$$=\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}(200-x)^2$$
$$=2x+2(200-x)^{2-1}\left[\frac{d}{dx}(200-x)\right]$$
$$=2x+2(200-x)(-1)$$
$$=2x+2x-400$$
$$=4x-400$$

20. $f(x)=x^2+(100-x)^2$

$$f'(x)=2x+2(100-x)^{2-1}(-1)$$
$$=2x+2x-200$$
$$=4x-200$$

21. $G(x)=\sqrt[3]{2x-1}+(4-x)^2=(2x-1)^{1/3}+(4-x)^2$

Using the Sum Rule and the Extended Power Rule, we have:

$$G'(x)=\frac{d}{dx}\left[(2x-1)^{1/3}+(4-x)^2\right]$$
$$=\frac{d}{dx}(2x-1)^{1/3}+\frac{d}{dx}(4-x)^2$$
$$=\frac{1}{3}(2x-1)^{1/3-1}\left(\frac{d}{dx}(2x-1)\right)+2(4-x)^{2-1}\left(\frac{d}{dx}(4-x)\right)$$
$$=\frac{1}{3}(2x-1)^{-2/3}(2)+2(4-x)^1(-1)$$
$$=\frac{2}{3}(2x-1)^{-2/3}-2(4-x)$$
$$=\frac{2}{3(2x-1)^{2/3}}-8+2x$$
$$=\frac{2}{3\sqrt[3]{(2x-1)^2}}-8+2x$$

22. $g(x)=\sqrt{x}+(x-3)^3=x^{1/2}+(x-3)^3$

$$g'(x)=\frac{d}{dx}\left[x^{1/2}+(x-3)^3\right]$$
$$=\frac{1}{2}x^{1/2-1}+3(x-3)^{3-1}(1)$$
$$=\frac{1}{2x^{1/2}}+3(x-3)^2$$
$$=\frac{1}{2\sqrt{x}}+3(x-3)^2$$

23. $f(x)=-5x(2x-3)^4$

Using the Product Rule, we have

$$f'(x)=\frac{d}{dx}\left[-5x(2x-3)^4\right]$$
$$=-5x\frac{d}{dx}\left[(2x-3)^4\right]+(2x-3)^4\frac{d}{dx}(-5x)$$

Using the Extended Power Rule, we have

$$f'(x)=-5x\left[4(2x-3)^3\left(\frac{d}{dx}(2x-3)\right)\right]+(2x-3)^4(-5)$$
$$=-5x\left[4(2x-3)^3(2)\right]+(2x-3)^4(-5)$$
$$=-40x(2x-3)^3-5(2x-3)^4$$
$$=-5(2x-3)^3\left[8x+(2x-3)\right] \quad \text{Factoring}$$
$$=-5(2x-3)^3(10x-3)$$

24. $f(x)=-3x(5x+4)^6$

$$f'(x)=-3x\left[6(5x+4)^{6-1}(5)\right]+(5x+4)^6(-3)$$
$$=-90x(5x+4)^5-3(5x+4)^6$$
$$=-3(5x+4)^5\left[30x+5x+4\right]$$
$$=-3(5x+4)^5(35x+4)$$

25. $F(x)=(5x+2)^4(2x-3)^8$

Using the Product Rule and the Extended Power Rule, we have

$$F'(x)=\frac{d}{dx}\left[(5x+2)^4(2x-3)^8\right]$$
$$=(5x+2)^4\frac{d}{dx}(2x-3)^8+(2x-3)^8\frac{d}{dx}(5x+2)^4$$
$$=(5x+2)^4\left[8(2x-3)^7(2)\right]+(2x-3)^8\left[4(5x+2)^3(5)\right]$$
$$=16(5x+2)^4(2x-3)^7+20(2x-3)^8(5x+2)^3$$
$$=4(5x+2)^3(2x-3)^7\left[4(5x+2)+5(2x-3)\right]$$
$$=4(5x+2)^3(2x-3)^7(30x-7)$$

26. $g(x)=(3x-1)^7(2x+1)^5$

$$g'(x)=\frac{d}{dx}\left[(3x-1)^7(2x+1)^5\right]$$
$$=(3x-1)^7\left[5(2x+1)^4(2)\right]+(2x+1)^5\left[7(3x-1)^6(3)\right]$$
$$=10(3x-1)^7(2x+1)^4+21(2x+1)^5(3x-1)^6$$
$$=(3x-1)^6(2x+1)^4\left[10(3x-1)+21(2x+1)\right]$$
$$=(3x-1)^6(2x+1)^4(72x+11)$$

27. $f(x)=x^2\sqrt{4x-1}=x^2(4x-1)^{1/2}$

Using the Product Rule and the Extended Power Rule, we have

$$f'(x)=\frac{d}{dx}\left[x^2(4x-1)^{1/2}\right]$$
$$=x^2\left[\frac{1}{2}(4x-1)^{-1/2}(4)\right]+(4x-1)^{1/2}(2x)$$
$$f'(x)=\frac{2x^2}{(4x-1)^{1/2}}+2x(4x-1)^{1/2}$$
$$=\frac{2x^2}{\sqrt{(4x-1)}}+2x\sqrt{(4x-1)}$$

The derivative can be simplified as follows:

$$f'(x)=\frac{2x^2}{\sqrt{4x-1}}+\frac{2x\sqrt{4x-1}}{1}\cdot\frac{\sqrt{4x-1}}{\sqrt{4x-1}}$$
$$=\frac{2x^2}{\sqrt{4x-1}}+\frac{2x(4x-1)}{\sqrt{4x-1}}$$
$$=\frac{2x^2}{\sqrt{4x-1}}+\frac{8x^2-2x}{\sqrt{4x-1}}$$
$$=\frac{10x^2-2x}{\sqrt{4x-1}}$$
$$=\frac{2x(5x-1)}{\sqrt{4x-1}}$$

28. $f(x)=x^3\sqrt{5x+2}=x^3(5x+2)^{1/2}$

$$f'(x)=x^3\left[\frac{1}{2}(5x+2)^{-1/2}(5)\right]+(5x+2)^{1/2}(3x^2)$$
$$=\frac{5}{2}x^3(5x+2)^{-1/2}+3x^2(5x+2)^{1/2}$$
$$=\frac{5x^3}{2\sqrt{5x+2}}+3x^2\sqrt{5x+2}$$

The previous derivative can be simplified as follows:

$$f'(x)=\frac{x^2(35x+12)}{2\sqrt{5x+2}}$$

29. $F(x)=\sqrt[4]{x^2-5x+2}=(x^2-5x+2)^{1/4}$

Using the Extended Power Rule, we have

$$F'(x)=\frac{d}{dx}(x^2-5x+2)^{1/4}$$
$$=\frac{1}{4}(x^2-5x+2)^{1/4-1}\cdot\frac{d}{dx}(x^2-5x+2)$$
$$=\frac{1}{4}(x^2-5x+2)^{-3/4}(2x-5)$$
$$=\frac{2x-5}{4(x^2-5x+2)^{3/4}}$$

30. $G(x)=\sqrt[3]{x^5+6x}=(x^5+6x)^{1/3}$

$$G'(x)=\frac{d}{dx}\left[(x^5+6x)^{1/3}\right]$$
$$=\frac{1}{3}(x^5+6x)^{-2/3}(5x^4+6)$$
$$=\frac{5x^4+6}{3(x^5+6x)^{2/3}}$$
$$=\frac{5x^4+6}{3\cdot\sqrt[3]{(x^5+6x)^2}}$$

31. $f(x)=\left(\frac{3x-1}{5x+2}\right)^4$

Using the Extended Power Rule, we have

$$f'(x)=4\left(\frac{3x-1}{5x+2}\right)^3\frac{d}{dx}\left[\frac{3x-1}{5x+2}\right]$$

Using the Quotient Rule, we have

$$f'(x)=4\left(\frac{3x-1}{5x+2}\right)^3\left[\frac{(5x+2)(3)-(3x-1)(5)}{(5x+2)^2}\right]$$
$$=4\left(\frac{3x-1}{5x+2}\right)^3\left[\frac{15x+6-15x+5}{(5x+2)^2}\right]$$
$$=4\left(\frac{3x-1}{5x+2}\right)^3\left[\frac{11}{(5x+2)^2}\right]$$
$$=\frac{44(3x-1)^3}{(5x+2)^5}$$

32. $f(x)=\left(\frac{2x}{x^2+1}\right)^3$

$$f'(x)=3\left(\frac{2x}{x^2+1}\right)^2\left[\frac{(x^2+1)(2)-2x(2x)}{(x^2+1)^2}\right]$$
$$=3\left(\frac{2x}{x^2+1}\right)^2\left[\frac{2x^2+2-4x^2}{(x^2+1)^2}\right]$$
$$=3\left(\frac{2x}{x^2+1}\right)^2\left[\frac{2-2x^2}{(x^2+1)^2}\right]$$
$$=\frac{3(2x)^2(2-2x^2)}{(x^2+1)^3}$$
$$=\frac{24x^2(1-x^2)}{(x^2+1)^3}$$
$$=\frac{-24x^2(x^2-1)}{(x^2+1)^3}$$

33. $g(x)=\sqrt{\frac{3+2x}{5-x}}=\left(\frac{3+2x}{5-x}\right)^{1/2}$

Using the Extended Power Rule, we have

$$g'(x)=\frac{d}{dx}\left(\left(\frac{3+2x}{5-x}\right)^{1/2}\right)$$
$$=\frac{1}{2}\left(\frac{3+2x}{5-x}\right)^{1/2-1}\cdot\frac{d}{dx}\left(\frac{3+2x}{5-x}\right)$$

Using the Quotient Rule, we have

$$=\frac{1}{2}\left(\frac{3+2x}{5-x}\right)^{-1/2}\left[\frac{(5-x)(2)-(3+2x)(-1)}{(5-x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{5-x}{3+2x}\right)^{1/2}\left[\frac{10-2x+3+2x}{(5-x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{5-x}{3+2x}\right)^{1/2}\left[\frac{13}{(5-x)^2}\right]$$
$$=\frac{13}{2(5-x)^{3/2}(3+2x)^{1/2}}$$
$$=\frac{13}{2\sqrt{(5-x)^3}\cdot\sqrt{(3+2x)}}$$

34. $g(x)=\sqrt{\frac{4-x}{3+x}}=\left(\frac{4-x}{3+x}\right)^{1/2}$

$$g'(x)=\frac{1}{2}\left(\frac{4-x}{3+x}\right)^{-1/2}\left[\frac{(3+x)(-1)-(4-x)(1)}{(3+x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{4-x}{3+x}\right)^{-1/2}\left[\frac{-3-x-4+x}{(3+x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{3+x}{4-x}\right)^{1/2}\left[\frac{-7}{(3+x)^2}\right]$$
$$=\frac{-7}{2(3+x)^{3/2}(4-x)^{1/2}}$$
$$=\frac{-7}{2\sqrt{(3+x)^3}\cdot\sqrt{4-x}}$$

35. $f(x)=(2x^3-3x^2+4x+1)^{100}$

Using the Extended Power Rule, we have

$$f'(x)=100(2x^3-3x^2+4x+1)^{99}(6x^2-6x+4)$$
$$=200(2x^3-3x^2+4x+1)^{99}(3x^2-3x+2)$$

36. $f(x)=(7x^4+6x^3-x)^{204}$

$$f'(x)=204(7x^4+6x^3-x)^{203}(28x^3+18x^2-1)$$

37. $h(x)=\left(\frac{1-3x}{2-7x}\right)^{-5}=\left(\frac{2-7x}{1-3x}\right)^5$

Using the Extended Power Rule, we have

$$h'(x)=\frac{d}{dx}\left[\left(\frac{2-7x}{1-3x}\right)^5\right]$$
$$=5\left(\frac{2-7x}{1-3x}\right)^{5-1}\left[\frac{d}{dx}\left(\frac{2-7x}{1-3x}\right)\right]$$

The solution is continued on the next page.

Next, using the Quotient Rule, we have

$h'(x)$

$$= 5\left(\frac{2-7x}{1-3x}\right)^4\left(\frac{(1-3x)(-7)-(2-7x)(-3)}{(1-3x)^2}\right)$$

$$= 5\left(\frac{2-7x}{1-3x}\right)^4\left(\frac{-7+21x+6-21x}{(1-3x)^2}\right)$$

$$= 5\left(\frac{2-7x}{1-3x}\right)^4\left(\frac{-1}{(1-3x)^2}\right)$$

$$= \frac{-5(2-7x)^4}{(1-3x)^6}$$

38. $g(x)=\left(\frac{2x+3}{5x-1}\right)^{-4}=\left(\frac{5x-1}{2x+3}\right)^4$

$$g'(x)=4\left(\frac{5x-1}{2x+3}\right)^3\left[\frac{(2x+3)(5)-(5x-1)(2)}{(2x+3)^2}\right]$$

$$=4\left(\frac{5x-1}{2x+3}\right)^3\left[\frac{10x+15-10x+2}{(2x+3)^2}\right]$$

$$=4\left(\frac{5x-1}{2x+3}\right)^3\left[\frac{17}{(2x+3)^2}\right]$$

$$=\frac{68(5x-1)^3}{(2x+3)^5}$$

39. $f(x)=\sqrt{\frac{x^2+x}{x^2-x}}=\left(\frac{x^2+x}{x^2-x}\right)^{1/2}$

Using the Extended Power Rule, we have

$$f'(x)=\frac{1}{2}\left(\frac{x^2+x}{x^2-x}\right)^{1/2-1}\cdot\frac{d}{dx}\left[\frac{x^2+x}{x^2-x}\right]$$

Using the Quotient Rule, we have

$f'(x)$

$$=\frac{1}{2}\left(\frac{x^2+x}{x^2-x}\right)^{-1/2}\left[\frac{(x^2-x)(2x+1)-(x^2+x)(2x-1)}{(x^2-x)^2}\right]$$

$$=\frac{1}{2}\left(\frac{x^2+x}{x^2-x}\right)^{-1/2}\left[\frac{2x^3-x^2-x-2x^3-x^2+x}{(x^2-x)^2}\right]$$

Continued at the top of the next column.

$$f'(x)=\frac{1}{2}\left(\frac{x^2-x}{x^2+x}\right)^{1/2}\left(\frac{-2x^2}{(x^2-x)^2}\right)$$

$$=\frac{-x^2}{(x^2-x)^{3/2}(x^2+x)^{1/2}}$$

The previous derivative can be simplified as follows:

$$f'(x)=\frac{-x^2}{(x^2-x)^{3/2}(x^2+x)^{1/2}}$$

$$=\frac{-x^2}{x^{3/2}(x-1)^{3/2}x^{1/2}(x+1)^{1/2}} \quad \text{Factoring}$$

$$=\frac{-x^2}{x^2(x-1)^{3/2}(x+1)^{1/2}}$$

$$=\frac{-1}{(x-1)^{3/2}(x+1)^{1/2}}$$

40. $f(x)=\sqrt[3]{\frac{4-x^3}{x-x^2}}=\left(\frac{4-x^3}{x-x^2}\right)^{1/3}$

$f'(x)$

$$=\frac{1}{3}\left(\frac{4-x^3}{x-x^2}\right)^{-2/3}\left[\frac{(x-x^2)(-3x^2)-(4-x^3)(1-2x)}{(x-x^2)^2}\right]$$

$$=\frac{1}{3}\left(\frac{x-x^2}{4-x^3}\right)^{2/3}\left[\frac{-3x^3+3x^4-4+8x+x^3-2x^4}{(x-x^2)^2}\right]$$

Next, we simplify the derivative.

$$f'(x)=\frac{1}{3}\left(\frac{x-x^2}{4-x^3}\right)^{2/3}\left[\frac{x^4-2x^3+8x-4}{(x-x^2)^2}\right]$$

$$=\frac{x^4-2x^3+8x-4}{3(4-x^3)^{2/3}(x-x^2)^{4/3}}$$

41. $f(x)=\dfrac{(5x-4)^7}{(6x+1)^3}$

Using the Quotient Rule and the Extended Power Rule, we have:

$$f'(x)=\frac{d}{dx}\left[\frac{(5x-4)^7}{(6x+1)^3}\right]$$

$$=\frac{(6x+1)^3\left[7(5x-4)^6(5)\right]-(5x-4)^7\left[3(6x+1)^2(6)\right]}{\left((6x+1)^3\right)^2}$$

Simplifying the expression, we have

$$f'(x)=\frac{35(5x-4)^6(6x+1)^3-18(5x-4)^7(6x+1)^2}{(6x+1)^6}$$

$$=\frac{(5x-4)^6(6x+1)^2\left[35(6x+1)-18(5x-4)\right]}{(6x+1)^6}$$

$$=\frac{(5x-4)^6\left[210x+35-90x+72\right]}{(6x+1)^4}$$

$$=\frac{(5x-4)^6\left[120x+107\right]}{(6x+1)^4}$$

Therefore,

$$f'(x)=\frac{(5x-4)^6(120x+107)}{(6x+1)^4}$$

42. $f(x)=\dfrac{(2x+3)^4}{(3x-2)^5}$

$$f'(x)=\frac{d}{dx}\left[\frac{(2x+3)^4}{(3x-2)^5}\right]$$

$$=\frac{(3x-2)^5\left[4(2x+3)^3(2)\right]-(2x+3)^4\left[5(3x-2)^4(3)\right]}{\left((3x-2)^5\right)^2}$$

$$=\frac{8(2x+3)^3(3x-2)^5-15(2x+3)^4(3x-2)^4}{(3x-2)^{10}}$$

$$=\frac{(2x+3)^3(3x-2)^4\left[8(3x-2)-15(2x+3)\right]}{(3x-2)^{10}}$$

$$=\frac{(2x+3)^3\left[-6x-61\right]}{(3x-2)^6}$$

Therefore,

$$f'(x)=\frac{-(2x+3)^3(6x+61)}{(3x-2)^6}$$

43. $f(x)=12(2x+1)^{2/3}(3x-4)^{5/4}$

Using the Product Rule and the Extended Power Rule, we have:

$$f'(x)=12\frac{d}{dx}\left[(2x+1)^{2/3}(3x-4)^{5/4}\right]$$

$$=12\left[(2x+1)^{2/3}\left[\frac{5}{4}(3x-4)^{1/4}(3)\right]\right]+12\left[(3x-4)^{5/4}\left[\frac{2}{3}(2x+1)^{-1/3}(2)\right]\right]$$

$$=12\left[\frac{15}{4}(2x+1)^{2/3}(3x-4)^{1/4}\right]+12\left[\frac{4}{3}(3x-4)^{5/4}(2x+1)^{-1/3}\right]$$

$$=45(2x+1)^{2/3}(3x-4)^{1/4}+\frac{16(3x-4)^{5/4}}{(2x+1)^{1/3}}$$

We simplify the derivative as follows:

$$f'(x)=\frac{45(2x+1)^{2/3}(3x-4)^{1/4}}{1}\cdot\frac{(2x+1)^{1/3}}{(2x+1)^{1/3}}+\frac{16(3x-4)^{5/4}}{(2x+1)^{1/3}}$$

$$=\frac{45(2x+1)(3x-4)^{1/4}}{(2x+1)^{1/3}}+\frac{16(3x-4)^{5/4}}{(2x+1)^{1/3}}$$

$$=\frac{45(2x+1)(3x-4)^{1/4}+16(3x-4)^{5/4}}{(2x+1)^{1/3}}$$

$$=\frac{(3x-4)^{1/4}\left[45(2x+1)+16(3x-4)^{4/4}\right]}{(2x+1)^{1/3}}$$

$$=\frac{(3x-4)^{1/4}\left[45(2x+1)+16(3x-4)\right]}{(2x+1)^{1/3}}$$

$$=\frac{(3x-4)^{1/4}\left[90x+45+48x-64\right]}{(2x+1)^{1/3}}$$

$$=\frac{(3x-4)^{1/4}\left[138x-19\right]}{(2x+1)^{1/3}}$$

Therefore,

$$f'(x)=\frac{(3x-4)^{1/4}(138x-19)}{(2x+1)^{1/3}}$$

44. $y = 6\sqrt[3]{x^2+x}\left(x^4-6x\right)^3$

$y = 6\left(x^2+x\right)^{1/3}\left(x^4-6x\right)^3$

$\frac{dy}{dx}$

$= 6\left[\left(x^2+x\right)^{1/3}\left[3\left(x^4-6x\right)^2\left(4x^3-6\right)\right]\right] +$

$6\left[\left(x^4-6x\right)^3\left[\frac{1}{3}\left(x^2+x\right)^{-2/3}(2x+1)\right]\right]$

$= 18\left(x^2+x\right)^{1/3}\left(x^4-6x\right)^2\left(4x^3-6\right) +$

$2\left(x^4-6x\right)^3\left(x^2+x\right)^{-2/3}(2x+1)$

45. $y = \frac{15}{u^3} = 15u^{-3}$ and $u = 2x+1$

$\frac{dy}{du} = 15\left(-3u^{-3-1}\right) = -45u^{-4} = \frac{-45}{u^4}$

$\frac{du}{dx} = 2$

Applying the Chain Rule, we have:

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

$= \frac{-45}{u^4}\cdot 2$

$= \frac{-45}{(2x+1)^4}\cdot 2$ Substituting $2x+1$ for u

$= \frac{-90}{(2x+1)^4}$ Simplifying

46. $y = \sqrt{u} = u^{1/2}$ and $u = x^2-1$

$\frac{dy}{du} = \frac{1}{2}u^{1/2-1} = \frac{1}{2}u^{-1/2} = \frac{1}{2\sqrt{u}}$

$\frac{du}{dx} = 2x^{2-1} = 2x$

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

$= \frac{1}{2\sqrt{u}}\cdot 2x$

$= \frac{x}{\sqrt{x^2-1}}$

47. $y = u^{50}$ and $u = 4x^3-2x^2$

$\frac{dy}{du} = 50u^{50-1} = 50u^{49}$

$\frac{du}{dx} = 4\left(3x^{3-1}\right) - 2\left(2x^{2-1}\right) = 12x^2-4x$

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

$= 50u^{49}\cdot\left(12x^2-4x\right)$

Substituting $4x^3-2x^2$ for u

$= 50\left(4x^3-2x^2\right)^{49}\cdot\left(12x^2-4x\right)$

$= 200x(3x-1)\left(4x^3-2x^2\right)^{49}$ Simplifying

48. $y = \frac{u+1}{u-1}$ and $u = 1+\sqrt{x} = 1+x^{1/2}$

$\frac{dy}{du} = \frac{(u-1)(1)-(u+1)(1)}{(u-1)^2} = \frac{-2}{(u-1)^2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

$= \frac{-2}{(u-1)^2}\cdot\frac{1}{2\sqrt{x}}$

$= \frac{-1}{\sqrt{x}\left(\left(1+\sqrt{x}\right)-1\right)^2}$ Substituting $1+\sqrt{x}$ for u.

$= \frac{-1}{\sqrt{x}\left(\sqrt{x}\right)^2}$ Simplifying

$= \frac{-1}{x^{3/2}}$

49. $y = (u+1)(u-1)$ and $u = x^3+1$

$\frac{dy}{du} = (u+1)(1)+(u-1)(1)$ Product Rule

$= 2u$

$\frac{du}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

$= (2u)\cdot\left(3x^2\right)$ Substituting x^3+1 for u

$= \left(2\left(x^3+1\right)\right)\cdot\left(3x^2\right)$

$= 6x^2\left(x^3+1\right)$ Simplifying

50. $y = u(u+1)$ and $u = x^3 - 2x$

$$\frac{dy}{du} = u(1) + (u+1)(1) = 2u + 1$$

$$\frac{du}{dx} = 3x^2 - 2$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (2u+1) \cdot (3x^2 - 2)$$

$$= \left(2(x^3 - 2x) + 1\right) \cdot (3x^2 - 2)$$

$$= (2x^3 - 4x + 1) \cdot (3x^2 - 2)$$

51. $y = 5u^2 + 3u$ where $u = x^3 + 1$

$$\frac{dy}{du} = 10u + 3$$

$$\frac{du}{dx} = 3x^2$$

Applying the Chain Rule, we have:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (10u + 3) \cdot (3x^2)$$

$$= \left(10(x^3 + 1) + 3\right) \cdot (3x^2) \quad \text{Substituting for } u$$

$$= 3x^2\left((10x^3 + 10) + 3\right)$$

$$= 3x^2(10x^3 + 13)$$

52. $y = u^3 - 7u^2$ where $u = x^2 + 3$

$$\frac{dy}{du} = 3u^2 - 14u$$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (3u^2 - 14u) \cdot (2x)$$

$$= \left(3(x^2 + 3)^2 - 14(x^2 + 3)\right) \cdot (2x)$$

$$= (3x^4 + 18x^2 + 27 - 14x^2 - 42)(2x)$$

$$= (3x^4 + 4x^2 - 15)(2x)$$

$$= 2x(x^2 + 3)(3x^2 - 5)$$

53. $y = \sqrt{7 - 3u} = (7 - 3u)^{1/2}$ where $u = x^2 - 9$

$$\frac{dy}{du} = \frac{1}{2}(7 - 3u)^{-1/2}(-3) \quad \text{Extended Power Rule}$$

$$= \frac{-3}{2(7 - 3u)^{1/2}}$$

$$\frac{du}{dx} = 2x$$

We apply the Chain Rule.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \left(\frac{-3}{2(7 - 3u)^{1/2}}\right) \cdot (2x)$$

$$= \left(\frac{-3}{2\left(7 - 3(x^2 - 9)\right)^{1/2}}\right) \cdot (2x) \quad \text{Substituting}$$

$$= \frac{-3x}{(34 - 3x^2)^{1/2}}$$

$$= \frac{-3x}{\sqrt{34 - 3x^2}}$$

54. $y = \sqrt[3]{2u + 5} = (2u + 5)^{1/3}$ where $u = x^2 - x$

$$\frac{dy}{du} = \frac{1}{3}(2u + 5)^{-2/3}(2) = \frac{2}{3(2u + 5)^{2/3}}$$

$$\frac{du}{dx} = 2x - 1$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \left(\frac{2}{3(2u + 5)^{2/3}}\right) \cdot (2x - 1)$$

$$= \left(\frac{2}{3\left(2(x^2 - x) + 5\right)^{2/3}}\right) \cdot (2x - 1)$$

$$= \frac{2(2x - 1)}{3(2x^2 - 2x + 5)^{2/3}}$$

55. $y=\dfrac{1}{u^2+u}$ and $u=5+3t$

$$\frac{dy}{du}=\frac{\left(u^2+u\right)(0)-(1)(2u+1)}{\left(u^2+u\right)^2}\quad\text{Quotient Rule}$$
$$=\frac{-(2u+1)}{\left(u^2+u\right)^2}$$
$$\frac{du}{dt}=3$$

We apply the Chain Rule.

$$\frac{dy}{dt}=\frac{dy}{du}\cdot\frac{du}{dt}$$
$$=\left(\frac{-(2u+1)}{\left(u^2+u\right)^2}\right)\cdot(3)$$
$$=\left(\frac{-(2(5+3t)+1)}{\left((5+3t)^2+(5+3t)\right)^2}\right)\cdot(3)\quad\text{Substituting}$$
$$=\frac{-3(10+6t+1)}{\left((5+3t)^2+(5+3t)\right)^2}$$
$$=\frac{-3(6t+11)}{(5+3t)^2\left((5+3t)+1\right)^2}\quad\text{Factoring}$$
$$=\frac{-3(6t+11)}{(5+3t)^2(6+3t)^2}$$

56. $y=\dfrac{1}{3u^5-7}$ and $u=7t^2+1$

$$\frac{dy}{du}=\frac{\left(3u^5-7\right)(0)-(1)\left(15u^4\right)}{\left(3u^5-7\right)^2}\quad\text{Quotient Rule}$$
$$=\frac{-15u^4}{\left(3u^5-7\right)^2}$$
$$\frac{du}{dt}=14t$$

We apply the Chain Rule at the top of the next column.

$$\frac{dy}{dt}=\frac{dy}{du}\cdot\frac{du}{dt}$$
$$=\left(\frac{-15u^4}{\left(3u^5-7\right)^2}\right)\cdot(14t)$$
$$=\left(\frac{-15\left(7t^2+1\right)^4}{\left(3\left(7t^2+1\right)^5-7\right)^2}\right)\cdot(14t)$$
$$=\frac{-210t\left(7t^2+1\right)^4}{\left(3\left(7t^2+1\right)^5-7\right)^2}$$

57. $y=\left(x^3-4x\right)^{10}$

First, we find the derivative using the Extended Power Rule.

$$\frac{dy}{dx}=10\left(x^3-4x\right)^9\left(3x^2-4\right)$$

When $x=2$,

$$\frac{dy}{dx}=10\left((2)^3-4(2)\right)^9\left(3(2)^2-4\right)$$
$$=10(0)^9(8)$$
$$=0$$

Thus, the slope of the tangent line at the point, $(2,0)$ is 0. The equation of the horizontal line passing through the point $(2,0)$ is $y=0$.

58. $y=\sqrt{x^2+3x}=\left(x^2+3x\right)^{1/2}$

$$\frac{dy}{dx}=\frac{1}{2}\left(x^2+3x\right)^{1/2-1}(2x+3)$$
$$=\frac{2x+3}{2\sqrt{x^2+3x}}$$

When $x=1$,

$$\frac{dy}{dx}=\frac{2(1)+3}{2\sqrt{(1)^2+3(1)}}=\frac{5}{2\sqrt{4}}=\frac{5}{2\cdot 2}=\frac{5}{4}$$

Thus, the slope of the tangent line at $(1,2)$ is $\frac{5}{4}$.

Using the point-slope equation, we find the equation of the tangent line on the next page.

Using the information on the previous page, we have:

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{5}{4}(x-1)$$
$$y - 2 = \frac{5}{4}x - \frac{5}{4}$$
$$y = \frac{5}{4}x + \frac{3}{4}$$

59. $y = x\sqrt{2x+3} = x(2x+3)^{1/2}$

First, we find the derivative using the Product Rule and the Extended Power Rule.

$$\frac{dy}{dx} = x\left(\frac{1}{2}(2x+3)^{1/2-1}(2)\right) + (2x+3)^{1/2}(1)$$
$$= \frac{x}{\sqrt{2x+3}} + \sqrt{2x+3}$$

When $x = 3$,

$$\frac{dy}{dx} = \frac{(3)}{\sqrt{2(3)+3}} + \sqrt{2(3)+3}$$
$$= \frac{3}{\sqrt{9}} + \sqrt{9}$$
$$= \frac{3}{3} + 3$$
$$= 1 + 3 = 4$$

Thus, the slope of the tangent line at $(3,9)$ is 4.

Using the point-slope equation, we find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 9 = 4(x-3)$$
$$y - 9 = 4x - 12$$
$$y = 4x - 3$$

60. $y = \left(\frac{2x+3}{x-1}\right)^3$

$$\frac{dy}{dx} = 3\left(\frac{2x+3}{x-1}\right)^2\left[\frac{(x-1)(2)-(2x+3)(1)}{(x-1)^2}\right]$$
$$= 3\left(\frac{2x+3}{x-1}\right)^2\left[\frac{2x-2-2x-3}{(x-1)^2}\right]$$
$$= 3\left(\frac{2x+3}{x-1}\right)^2\left[\frac{-5}{(x-1)^2}\right]$$
$$= \frac{-15(2x+3)^2}{(x-1)^4}$$

When $x = 2$,

$$\frac{dy}{dx} = \frac{-15(2(2)+3)^2}{((2)-1)^4}$$
$$= \frac{-15(7)^2}{(1)^4}$$
$$= -15 \cdot 49$$
$$= -735$$

Thus, the slope of the tangent line at $(2,343)$ is -735.

Using the point-slope equation, we find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 343 = -735(x-2)$$
$$y - 343 = -735x + 1470$$
$$y = -735x + 1813$$

61. $g(x) = \left(\frac{6x+1}{2x-5}\right)^2$

a) Using Extended Power Rule and the Quotient Rule, we have

$$g'(x) = 2\left(\frac{6x+1}{2x-5}\right)^{2-1} \cdot \frac{d}{dx}\left(\frac{6x+1}{2x-5}\right)$$
$$= 2\left(\frac{6x+1}{2x-5}\right)\left[\frac{(2x-5)(6)-(6x+1)(2)}{(2x-5)^2}\right]$$
$$= \frac{2(6x+1)}{2x-5}\left[\frac{12x-30-12x-2}{(2x-5)^2}\right]$$
$$= \frac{2(6x+1)}{2x-5}\left[\frac{-32}{(2x-5)^2}\right]$$
$$= \frac{-64(6x+1)}{(2x-5)^3}$$

b) Using the Quotient Rule on

$g(x)=\dfrac{36x^2+12x+1}{4x^2-20x+25}$, we have

$$g'(x)=\frac{(4x^2-20x+25)(72x+12)}{(4x^2-20x+25)^2}-\frac{(36x^2+12x+1)(8x-20)}{(4x^2-20x+25)^2}$$

$$=\frac{288x^3-1392x^2+1560x+300}{(4x^2-20x+25)^2}-\frac{288x^3-624x^2-232x-20}{(4x^2-20x+25)^2}$$

$$=\frac{-768x^2+1792x+320}{(4x^2-20x+25)^2}$$

$$=\frac{-64(2x-5)(6x+1)}{(2x-5)^4}$$

$$=\frac{-64(6x+1)}{(2x-5)^3}$$

c) The results are the same. Which method is easier depends on the student. We believe that the Extended Power rule offers us a more efficient approach. It takes too much time to expand the function, and then factor it back to binomials.

62. $f(x)=\dfrac{x^2}{(1+x)^5}$

a) Using the Quotient Rule and the Extended Power Rule, we have:

$$f'(x)=\frac{(1+x)^5(2x)-x^2\left[5(1+x)^4(1)\right]}{\left((1+x)^5\right)^2}$$

$$=\frac{2x(1+x)^5-5x^2(1+x)^4}{(1+x)^{10}}$$

$$=\frac{(1+x)^4\left(2x(1+x)-5x^2\right)}{(1+x)^{10}}$$

$$=\frac{2x+2x^2-5x^2}{(1+x)^6}$$

$$=\frac{2x-3x^2}{(1+x)^6}=\frac{x(2-3x)}{(1+x)^6}$$

b) Using the Product Rule and the Extended Power Rule on $f(x)=x^2(1+x)^{-5}$, we have

$$f'(x)=x^2\left(-5(1+x)^{-6}(1)\right)+(1+x)^{-5}(2x)$$

$$=\frac{-5x^2}{(1+x)^6}+\frac{2x}{(1+x)^5}$$

$$=\frac{-5x^2}{(1+x)^6}+\frac{2x}{(1+x)^5}\cdot\frac{(1+x)}{(1+x)}$$

$$=\frac{-5x^2+2x+2x^2}{(1+x)^6}$$

$$=\frac{2x-3x^2}{(1+x)^6}$$

$$=\frac{x(2-3x)}{(1+x)^6}$$

c) The results are the same.

63. Using the Chain Rule:

$f(u)=u^3,\ g(x)=u=2x^4+1$

First find $f'(u)$ and $g'(x)$.

$f'(u)=3u^2$

$f'(g(x))=3(2x^4+1)^2$ Substituting $g(x)$ for u

$g'(x)=8x^3$

The Chain Rule states

$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$

Substituting, we have:

$$(f\circ g)'(x)=3(2x^4+1)^2\cdot(8x^3)$$

$$=24x^3(2x^4+1)^2$$

Therefore,

$$(f\circ g)'(-1)=24(-1)^3\left(2(-1)^4+1\right)^2$$

$$=-24(2+1)^2$$

$$=-24\cdot 9=-216$$

Alternatively, finding $f(g(x))$ first, we have:

$$f(g(x))=f(2x^4+1)=(2x^4+1)^3$$

By the Extended Power Rule:

$$f'(g(x))=3(2x^4+1)^2(8x^3)$$

$$=24x^3(2x^4+1)^2$$

Therefore, $f'(g(-1))=-216$ as above.

64. Using the Chain Rule:

$$f(u)=\frac{u+1}{u-1},\ g(x)=u=\sqrt{x}=x^{1/2}$$

$$f'(u)=\frac{(u-1)(1)-(u+1)(1)}{(u-1)^2}=\frac{-2}{(u-1)^2}$$

$$f'(g(x))=\frac{-2}{\left(\sqrt{x}-1\right)^2}$$

$$g'(x)=\frac{1}{2\sqrt{x}}$$

$$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$$

$$=\frac{-2}{\left(\sqrt{x}-1\right)^2}\cdot\frac{1}{2\sqrt{x}}$$

$$=\frac{-1}{\sqrt{x}\left(\sqrt{x}-1\right)^2}$$

Therefore,

$$(f\circ g)'(4)=\frac{-1}{\sqrt{4}\left(\sqrt{4}-1\right)^2}=\frac{-1}{2(1)^2}=-\frac{1}{2}.$$

Alternatively, finding $f(g(x))$ first, we have:

$$f(g(x))=f\left(\sqrt{x}\right)=\frac{\sqrt{x}+1}{\sqrt{x}-1}$$

Using the Quotient Rule:

$$f'(g(x))=\frac{\left(\sqrt{x}-1\right)\left(\frac{1}{2}x^{-1/2}\right)-\left(\sqrt{x}+1\right)\frac{1}{2}x^{-1/2}}{\left(\sqrt{x}-1\right)^2}$$

$$=\frac{\frac{1}{2}-\frac{1}{2}x^{-1/2}-\frac{1}{2}-\frac{1}{2}x^{-1/2}}{\left(\sqrt{x}-1\right)^2}$$

$$=\frac{-1}{\sqrt{x}\left(\sqrt{x}-1\right)^2}$$

Therefore, $(f\circ g)'(4)=-\frac{1}{2}$ as above.

65. Using the Chain Rule:

$$f(u)=\sqrt[3]{u}=u^{1/3},\ g(x)=u=1+3x^2$$

First find $f'(u)$ and $g'(x)$ at the top of the next column.

$$f'(u)=\frac{1}{3}u^{-2/3}=\frac{1}{3\cdot\sqrt[3]{u^2}}$$

$$f'(g(x))=\frac{1}{3\cdot\sqrt[3]{\left(1+3x^2\right)^2}}\quad \text{Substituting } g(x) \text{ for } u$$

$$g'(x)=6x$$

The Chain Rule states

$$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$$

Substituting, we have:

$$(f\circ g)'(x)=\frac{1}{3\cdot\sqrt[3]{\left(1+3x^2\right)^2}}\cdot(6x)$$

$$=\frac{2x}{\sqrt[3]{\left(1+3x^2\right)^2}}$$

Therefore,

$$(f\circ g)'(2)=\frac{2(2)}{\sqrt[3]{\left(1+3(2)^2\right)^2}}$$

$$=\frac{4}{\sqrt[3]{(13)^2}}$$

$$\approx 0.72348760$$

Alternatively, finding $f(g(x))$ first, we have:

$$f(g(x))=f\left(1+3x^2\right)=\left(1+3x^2\right)^{1/3}$$

By the Extended Power Rule:

$$f'(g(x))=\frac{1}{3}\left(1+3x^2\right)^{-2/3}(6x)$$

$$=\frac{2x}{\left(1+3x^2\right)^{2/3}}$$

Therefore $(f\circ g)'(2)=\frac{4}{\sqrt[3]{(13)^2}}\approx 0.72348760$.

66. Using the Chain Rule:

$$f(u)=2u^5,\ g(x)=u=\frac{3-x}{4+x}$$

$$f'(u)=10u^4$$

$$f'(g(x))=10\left(\frac{3-x}{4+x}\right)^4$$

$$g'(x)=\frac{(4+x)(-1)-(3-x)(1)}{(4+x)^2}$$

$$=\frac{-7}{(4+x)^2}$$

The solution is continued on the next page.

Using the information on the previous page, we have:

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

$$= 10\left(\frac{3-x}{4+x}\right)^4 \cdot \left(\frac{-7}{(4+x)^2}\right)$$

$$= \frac{-70(3-x)^4}{(4+x)^6}$$

Therefore,

$$(f \circ g)'(-10) = \frac{-70(3-(-10))^4}{(4+(-10))^6}$$

$$= \frac{-70(13)^4}{(-6)^6}$$

$$= -42.85129458$$

Alternatively, finding $f(g(x))$ first, we have:

$$f(g(x)) = f\left(\frac{3-x}{4+x}\right) = 2\left(\frac{3-x}{4+x}\right)^5$$

Using the Extended Power Rule:

$$f'(g(x)) = 2 \cdot 5\left(\frac{3-x}{4+x}\right)^4\left[\frac{(4+x)(-1)-(3-x)(1)}{(4+x)^2}\right]$$

$$= 10\left(\frac{3-x}{4+x}\right)^4\left[\frac{-7}{(4+x)^2}\right]$$

$$f'(g(x)) = \frac{-70(3-x)^4}{(4+x)^6}$$

Therefore, $(f \circ g)'(-10) = -42.85129458$ as above.

67. $f(x) = \left(2x^3 + (4x-5)^2\right)^6$

Letting $u = 2x^3 + (4x-5)^2$ and applying the Chain Rule, we have:

$f'(x)$

$$= 6\left(2x^3 + (4x-5)^2\right)^{6-1} \cdot \frac{d}{dx}\left(2x^3 + (4x-5)^2\right)$$

We will have to apply the chain rule again to find $\frac{d}{dx}\left(2x^3 + (4x-5)^2\right)$.

Applying the chain rule again, we have:

$$\frac{d}{dx}\left(2x^3 + (4x-5)^2\right)$$

$$= 2\frac{d}{dx}x^3 + \frac{d}{dx}(4x-5)^2$$

$$= 2\left(3x^2\right) + 2(4x-5)^{2-1} \cdot \frac{d}{dx}(4x-5)$$

$$= 6x^2 + 2(4x-5) \cdot 4$$

$$= 6x^2 + 32x - 40.$$

Therefore, the derivative is:

$$f'(x) = 6\left(2x^3 + (4x-5)^2\right)^5\left(6x^2 + 32x - 40\right).$$

68. $f(x) = \left(-x^5 + 4x + \sqrt{2x+1}\right)^3$

$f'(x)$

$$= 3\left(-x^5 + 4x + \sqrt{2x+1}\right)^2 \cdot \frac{d}{dx}\left(-x^5 + 4x + \sqrt{2x+1}\right)$$

Applying the chain rule again, we have:

$$\frac{d}{dx}\left(-x^5 + 4x + \sqrt{2x+1}\right)$$

$$= -5x^4 + 4 + \frac{1}{2\sqrt{2x+1}}\frac{d}{dx}(2x+1)$$

$$= -5x^4 + 4 + \frac{1}{\sqrt{2x+1}}.$$

Therefore the derivative is:

$$f'(x) = 3\left(-x^5 + 4x + \sqrt{2x+1}\right)^2 \cdot \left(-5x^4 + 4 + \frac{1}{\sqrt{2x+1}}\right)$$

69. $f(x) = \sqrt{x^2 + \sqrt{1-3x}} = \left(x^2 + (1-3x)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

Applying the Chain Rule, we have

$f'(x)$

$$= \frac{1}{2}\left(x^2 + (1-3x)^{\frac{1}{2}}\right)^{\frac{1}{2}-1} \cdot \frac{d}{dx}\left(x^2 + (1-3x)^{\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(x^2 + (1-3x)^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot \frac{d}{dx}\left(x^2 + (1-3x)^{\frac{1}{2}}\right)$$

The solution is continued on the next page.

Applying the chain rule again, we have

$$\frac{d}{dx}\left(x^2+(1-3x)^{\frac{1}{2}}\right)$$
$$=\frac{d}{dx}x^2+\frac{d}{dx}(1-3x)^{\frac{1}{2}}$$
$$=2x^{2-1}+\frac{1}{2}(1-3x)^{\frac{1}{2}-1}\cdot\frac{d}{dx}(1-3x)$$
$$=2x+\frac{1}{2\sqrt{1-3x}}\cdot(-3)$$
$$=2x-\frac{3}{2\sqrt{1-3x}}.$$

Therefore, the derivative is:

$$f'(x)$$
$$=\frac{1}{2}\left(x^2+(1-3x)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(2x-\frac{3}{2\sqrt{1-3x}}\right)$$
$$=\frac{1}{2\sqrt{x^2+\sqrt{1-3x}}}\left(2x-\frac{3}{2\sqrt{1-3x}}\right).$$

70. $f(x)=\sqrt[3]{2x+\left(x^2+x\right)^4}$

Applying the Chain Rule, we have

$$f'(x)$$
$$=\frac{1}{3}\left(2x+\left(x^2+x\right)^4\right)^{-\frac{2}{3}}\cdot\frac{d}{dx}\left(2x+\left(x^2+x\right)^4\right)$$

Applying the chain rule again, we have

$$\frac{d}{dx}\left(2x+\left(x^2+x\right)^4\right)$$
$$=2+4\left(x^2+x\right)^3\cdot\frac{d}{dx}\left(x^2+x\right)$$
$$=2+4\left(x^2+x\right)^3(2x+1)$$

Therefore, the derivative is:

$$f'(x)=\frac{1}{3}\left(2x+\left(x^2+x\right)^4\right)^{-\frac{2}{3}}\cdot\left(2+4\left(x^2+x\right)^3(2x+1)\right)$$

71. $R(x)=1000\sqrt{x^2-0.1x}=1000\left(x^2-0.1x\right)^{1/2}$

Using the Extended Power Rule, we have

$$R'(x)=1000\left[\frac{1}{2}\left(x^2-0.1x\right)^{-1/2}(2x-0.1)\right]$$
$$=500\left[\frac{2x-0.1}{\left(x^2-0.1x\right)^{1/2}}\right]$$
$$=\frac{500(2x-0.1)}{\sqrt{x^2-0.1x}}$$

Substituting 20 for x, we have

$$R'(20)=\frac{500(2(20)-0.1)}{\sqrt{(20)^2-0.1(20)}}$$
$$=1000.00314070$$
$$\approx 1000$$

When 20 planes have been sold, revenue is changing at a rate of 1,000 thousand of dollars per plane, or 1,000,000 dollars per plane.

72. $C(x)=2000\left(x^2+2\right)^{1/3}+700$

$$C'(x)=2000\left[\frac{1}{3}\left(x^2+2\right)^{-2/3}(2x)\right]$$
$$=\frac{4000x}{3}\left(x^2+2\right)^{-2/3}$$
$$=\frac{4000x}{3\left(x^2+2\right)^{2/3}}$$
$$C'(20)=\frac{4000(20)}{3\left((20)^2+2\right)^{2/3}}\approx 489.57364458$$

When 20 planes are produced, the total cost is changing at a rate of 489.574 thousand of dollars per plane, or 489,574 dollars per plane.

73. $P(x)=R(x)-C(x)$ and

$P'(x)=R'(x)-C'(x)$

Since we are trying to find the rate at which total profit is changing as a function of x, we can use the derivatives found in Exercise 71 and 72 to find the derivative of the profit function. There is no need to find the Profit function first and then take the derivative.

$$P'(x)=R'(x)-C'(x)$$
$$=\frac{500(2x-0.1)}{\sqrt{x^2-0.1x}}-\frac{4000x}{3\left(x^2+2\right)^{2/3}}$$

74. $C(x)=\sqrt{5x^2+60}$ and $x(t)=20t+40$. To find the rate of change of the cost function with respect to the number of months, we must apply the Chain Rule.

$$C'(x)=\frac{1}{2}\left(5x^2+60\right)^{-1/2}(10x)=5x\left(5x^2+60\right)^{-1/2}$$

$$C'(x(t))=5(20t+40)\left(5(20t+40)^2+60\right)^{-1/2}$$

$$x'(t)=20$$

Using the Chain Rule, we have:

$$\frac{dC}{dt}=C'(x(t))\cdot x'(t)$$
$$=5(20t+40)\left(5(20t+40)^2+60\right)^{-1/2}\cdot(20)$$
$$=100(20t+40)\left(5(20t+40)^2+60\right)^{-1/2}$$
$$=\frac{100(20t+40)}{\left(5(20t+40)^2+60\right)^{1/2}}$$

Substituting 4 in for *t*, we have:

$$\frac{dC}{dt}=\frac{100(20(4)+40)}{\left(5(20(4)+40)^2+60\right)^{1/2}}$$
$$\approx 44.70273729$$

After 4 months, the total cost is changing at a rate of 44.7 thousand of dollars per month, or 44,700 dollars per month.

75. Let *x* be the number of years since 2008.

$$C(x)=9.26x^4-85.27x^3+287.24x^2-309.12x+2651.4$$

a) Taking the derivative with respect to *x,* we have

$$\frac{dC}{dx}=9.26\left(4x^3\right)-85.27\left(3x^2\right)+287.24(2x)-309.12$$

$$\frac{dC}{dx}=37.04x^3-255.81x^2+574.48x-309.12$$

b) ✎Answers will vary. $\frac{dC}{dx}$ represents the rate at which consumer credit is changing with respect to time.

c) Substitute 6 for *x*. $[2014-2008=6]$

$$\frac{dC}{dx}=37.04(6)^3-255.81(6)^2+574.48(6)-309.12$$
$$=8000.64-9209.16+3446.88-309.12$$
$$=1929.24$$

Outstanding consumer credit will be rising approximately at a rate of 1929.24 billion of dollars per year in 2014.

76. $U(x)=80\sqrt{\frac{2x+1}{3x+4}}=80\left(\frac{2x+1}{3x+4}\right)^{1/2}$

$$U'(x)=40\left(\frac{2x+1}{3x+4}\right)^{-1/2}\cdot\left[\frac{(3x+4)(2)-(2x+1)(3)}{(3x+4)^2}\right]$$
$$=40\left(\frac{3x+4}{2x+1}\right)^{1/2}\left[\frac{5}{(3x+4)^2}\right]$$
$$=\frac{200}{(2x+1)^{1/2}(3x+4)^{3/2}}$$

77. $A=1000\left(1+\frac{r}{4}\right)^{20}$

a) Using the Extended Power Rule, we have

$$\frac{dA}{dr}=1000(20)\left(1+\frac{r}{4}\right)^{19}\left(\frac{1}{4}\right)$$
$$=5000\left(1+\frac{r}{4}\right)^{19}$$

b) ✎ $\frac{dA}{dr}$ is the rate at which the amount in the account is growing five years after it was invested with respect to the interest rate.

78. $A=1000(1+r)^3$

a) Using the Extended Power Rule, we have

$$\frac{dA}{dr}=1000(3)(1+r)^2(1)$$
$$=3000(1+r)^2$$

b) ✎ $\frac{dA}{dr}$ is the rate at which the amount in the account is growing three years after it was invested with respect to the interest rate.

79. $P(x)=0.08x^2+80x$ and $x=5t+1$

a) Substituting $5t+1$ for *x*, we have

$$P(t)=0.08(5t+1)^2+80(5t+1)$$
$$=0.08\left(25t^2+10t+1\right)+400t+80$$
$$=2t^2+400.8t+80.08$$

b) First we find the derivative with respect to *t.*

$$P'(t)=\frac{dP}{dt}\left(2t^2+400.8t+80.08\right)$$
$$=4t+400.8$$

Substituting 48 in for *t,* we have

$P'(48)=4(48)+400.8=592.80.$

After 48 months, profit is increasing at rate of 592.80 dollars per month.

80. $D(p)=\frac{80,000}{p}$ and $p=1.6t+9$

a) Substitute $1.6t+9$ in for *p* in the demand function.

$$D(t)=\frac{80,000}{1.6t+9}$$

b) Using the Quotient Rule, we have

$$D'(t)=\frac{(1.6t+9)(0)-(80,000)(1.6)}{(1.6t+9)^2}$$
$$=\frac{-128,000}{(1.6t+9)^2}$$

Substituting 100 for *t* into the derivative, we have:

$$D'(100)=\frac{-128,000}{\left(1.6(100)+9\right)^2}$$
$$=\frac{-128,000}{28,561}$$
$$\approx -4.48163580$$

After 100 days, quantity demanded is changing -4.482 units per day.

81. $D=0.85A(c+25)$ and $c=(140-y)\frac{w}{72x}$

a) Substituting 5 for *A* we have:

$$D(c)=0.85(5)(c+25)$$
$$=4.25(c+25)$$
$$=4.25c+106.25$$

Substituting 0.6 for *x,* 45 for *y*, we have:

$$c(w)=(140-45)\frac{w}{72(0.6)}$$
$$=95\frac{w}{43.2}$$
$$=\frac{95w}{43.2}\approx 2.199w$$

b) $\frac{dD}{dc}=\frac{d}{dc}(4.25c+106.25)=4.25$

The dosage changes at a rate of 4.25 mg per unit of creatine clearance.

c) $\frac{dc}{dw}=\frac{d}{dw}(2.199w)=2.199$

The creatine clearance changes at a rate of 2.199 unit of creatine clearance per kilogram.

d) By the Chain Rule:

$$\frac{dD}{dw}=\frac{dD}{dc}\cdot\frac{dc}{dw}=(4.25)(2.199)\approx 9.346$$

The dosage changes at a rate of 9.35 milligrams per kilogram.

e) ✎ Answers will vary. $\frac{dD}{dw}$ represents the rate of change of the dosage with respect to the patient's weight. For each additional kilogram of weight, the dosage is increased by about 9.35 milligrams.

82. $f(x)=x^2+1$

Note that $f'(x)=2x$ and

$f'(f(x))=2\left(x^2+1\right).$

Applying the Chain Rule to the iterated function, we have

$$\frac{d}{dx}\left[(f\circ f)(x)\right]=\frac{d}{dx}\left[f(f(x))\right]$$
$$=f'(f(x))\cdot f'(x)$$
$$=2\left(x^2+1\right)\cdot 2x$$
$$=4x^3+4x.$$

83. $f(x)=x+\sqrt{x}$

Note that $f'(x)=1+\frac{1}{2\sqrt{x}}$ and

$$f'(f(x))=1+\frac{1}{2\sqrt{x+\sqrt{x}}}.$$

The solution is continued on the next page.

Applying the Chain Rule to the iterated Function on the previous page, we have

$$\frac{d}{dx}\left[(f\circ f)(x)\right]$$
$$=\frac{d}{dx}\left[f(f(x))\right]$$
$$=f'(f(x))\cdot f'(x)$$
$$=\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}\right]\cdot\left[1+\frac{1}{2\sqrt{x}}\right]$$
$$=1+\frac{1}{2\sqrt{x}}+\left(\frac{1}{2\sqrt{x+\sqrt{x}}}\right)\left(1+\frac{1}{2\sqrt{x}}\right)$$

84. $f(x)=x^2+1$

Note that $f'(x)=2x$,

$f'(f(x))=2(x^2+1)$, and

$f'(f(f(x)))=2\left((x^2+1)^2+1\right)$

Applying the Chain Rule to the iterated function, we have

$$\frac{d}{dx}\left[(f\circ f\circ f)(x)\right]$$
$$=\frac{d}{dx}\left[f(f(f(x)))\right]$$
$$=f'(f(f(x)))\cdot f'(f(x))\cdot f'(x)$$
$$=2\left((x^2+1)^2+1\right)\cdot 2(x^2+1)\cdot 2x.$$

85. $f(x)=\sqrt{3x}=(3x)^{\frac{1}{2}}$

Note that

$$f'(x)=\frac{1}{2}(3x)^{-\frac{1}{2}}\bullet 3=\frac{3}{2}(3x)^{-\frac{1}{2}}=\frac{3^{\frac{1}{2}}}{2x^{\frac{1}{2}}}=\frac{1}{2}\bullet\left(\frac{3}{x}\right)^{\frac{1}{2}},$$

$$f'(f(x))=\frac{1}{2}\bullet\left(\frac{3}{(3x)^{\frac{1}{2}}}\right)^{\frac{1}{2}}$$
$$=\frac{1}{2}\bullet\left(\frac{3^{\frac{1}{2}}}{x^{\frac{1}{2}}}\right)^{\frac{1}{2}}$$
$$=\frac{1}{2}\bullet\left(\frac{3}{x}\right)^{\frac{1}{4}}$$

Now note that:

$$f(f(x))=\left(3(3x)^{\frac{1}{2}}\right)^{\frac{1}{2}}=3^{\frac{3}{4}}x^{\frac{1}{4}}$$

Therefore,

$$f'(f(f(x)))=\frac{1}{2}\bullet\left(\frac{3}{3^{\frac{3}{4}}x^{\frac{1}{4}}}\right)^{\frac{1}{2}}$$
$$=\frac{1}{2}\bullet\left(\frac{3^{\frac{1}{4}}}{x^{\frac{1}{4}}}\right)^{\frac{1}{2}}$$
$$=\frac{1}{2}\bullet\left(\frac{3}{x}\right)^{\frac{1}{8}}$$

Applying the Chain Rule to the iterated function on the above, we have

$$\frac{d}{dx}\left[(f\circ f\circ f)(x)\right]$$
$$=\frac{d}{dx}\left[f(f(f(x)))\right]$$
$$=f'(f(f(x)))\cdot f'(f(x))\cdot f'(x)$$
$$=\frac{1}{2}\left(\frac{3}{x}\right)^{\frac{1}{8}}\cdot\frac{1}{2}\left(\frac{3}{x}\right)^{\frac{1}{4}}\cdot\frac{1}{2}\left(\frac{3}{x}\right)^{\frac{1}{2}}$$
$$=\frac{1}{8}\left(\frac{3}{x}\right)^{\frac{1}{8}+\frac{1}{4}+\frac{1}{2}}=\frac{1}{8}\left(\frac{3}{x}\right)^{\frac{7}{8}}.$$

Looking at the solution, we do see a pattern in the composite derivatives. To find the derivative of n iterative compositions we would have the shortcut of

$$\frac{d}{dx}\left[\left(\underbrace{f\circ f\circ\cdots\circ f}_{n-\text{iterations}}\right)(x)\right]=\frac{1}{2^n}\left(\frac{3}{x}\right)^{\frac{2^n-1}{2^n}}.$$

86. $y=\sqrt{(2x-3)^2+1}=\left((2x-3)^2+1\right)^{1/2}$

$$\frac{dy}{dx}=\frac{1}{2}\left((2x-3)^2+1\right)^{-1/2}\left(2(2x-3)(2)\right)$$
$$=\frac{2(2x-3)}{\left((2x-3)^2+1\right)^{1/2}}$$
$$=\frac{2(2x-3)}{\sqrt{(2x-3)^2+1}}$$

87. $y=\sqrt[3]{x^3+6x+1}\cdot x^5=\left(x^3+6x+1\right)^{1/3}\cdot x^5$

Using the Product Rule and the Extended Power Rule, we have

The solution is continued on the next page.

$$\frac{dy}{dx} = \left(x^3+6x+1\right)^{1/3}\cdot\left(5x^4\right) + x^5\left[\frac{1}{3}\left(x^3+6x+1\right)^{-2/3}\left(3x^2+6\right)\right]$$

$$= 5x^4\left(x^3+6x+1\right)^{1/3} + \frac{3x^5\left(x^2+2\right)}{3\left(x^3+6x+1\right)^{2/3}}$$

The derivative can be further simplified by finding a common denominator and combining the fractions.

$$\frac{dy}{dx} = \frac{\left(x^3+6x+1\right)^{2/3}}{\left(x^3+6x+1\right)^{2/3}}\cdot\frac{5x^4\left(x^3+6x+1\right)^{1/3}}{1} + \frac{x^5\left(x^2+2\right)}{\left(x^3+6x+1\right)^{2/3}}$$

Continuing to simplify the derivative, we have:

$$\frac{dy}{dx} = \frac{5x^4\left(x^3+6x+1\right)}{\left(x^3+6x+1\right)^{2/3}} + \frac{x^5\left(x^2+2\right)}{\left(x^3+6x+1\right)^{2/3}}$$

$$= \frac{5x^7+30x^5+5x^4+x^7+2x^5}{\left(x^3+6x+1\right)^{2/3}}$$

$$= \frac{6x^7+32x^5+5x^4}{\left(x^3+6x+1\right)^{2/3}}$$

88. $y = \left(\frac{x}{\sqrt{x-1}}\right)^3$

$$\frac{dy}{dx} = 3\left(\frac{x}{\sqrt{x-1}}\right)^2\cdot\left(\frac{(x-1)^{1/2}(1) - x\left(\frac{1}{2}(x-1)^{-1/2}\cdot 1\right)}{\left(\sqrt{x-1}\right)^2}\right)$$

$$= \frac{3x^2}{x-1}\left(\frac{(x-1)^{1/2} - \frac{x}{2(x-1)^{1/2}}}{x-1}\right)$$

$$= \frac{3x^2}{x-1}\left(\frac{\frac{2x-2}{2(x-1)^{1/2}} - \frac{x}{2(x-1)^{1/2}}}{x-1}\right)$$

$$= \frac{3x^2}{(x-1)}\left(\frac{x-2}{2(x-1)^{3/2}}\right)$$

$$= \frac{3x^2(x-2)}{2(x-1)^{5/2}}$$

89. $y = \left(x\sqrt{1+x^2}\right)^3 = \left(x\left(1+x^2\right)^{1/2}\right)^3$

Using the Extended Power Rule and the Product Rule, we find the derivative:

$$\frac{dy}{dx} = \frac{d}{dx}\left(x\sqrt{1+x^2}\right)^3$$

$$= 3\left(x\left(1+x^2\right)^{1/2}\right)^2\cdot\left(x\left(\frac{1}{2}\left(1+x^2\right)^{-1/2}\cdot 2x\right) + \left(1+x^2\right)^{1/2}\cdot 1\right)$$

$$= 3x^2\left(1+x^2\right)\cdot\left(\frac{x^2}{\left(1+x^2\right)^{1/2}} + \left(1+x^2\right)^{1/2}\right)$$

$$= 3x^2\left(1+x^2\right)\cdot\left(\frac{x^2}{\left(1+x^2\right)^{1/2}} + \frac{1+x^2}{\left(1+x^2\right)^{1/2}}\right)$$

$$= 3x^2\left(1+x^2\right)\cdot\left(\frac{1+2x^2}{\left(1+x^2\right)^{1/2}}\right)$$

$$= \left(3x^2+6x^4\right)\left(1+x^2\right)^{1/2}$$

$$= \left(3x^2+6x^4\right)\sqrt{1+x^2}$$

90. $y = \frac{\sqrt{1-x^2}}{1-x} = \frac{\left(1-x^2\right)^{1/2}}{1-x}$

$$\frac{dy}{dx} = \frac{(1-x)\left(\frac{1}{2}\left(1-x^2\right)^{-1/2}(-2x)\right) - \left(1-x^2\right)^{1/2}(-1)}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{\frac{x(x-1)}{\left(1-x^2\right)^{1/2}} + \left(1-x^2\right)^{1/2}}{(1-x)^2}$$

$$= \frac{\frac{x^2-x}{\left(1-x^2\right)^{1/2}} + \frac{\left(1-x^2\right)^{1/2}}{1}\cdot\frac{\left(1-x^2\right)^{1/2}}{\left(1-x^2\right)^{1/2}}}{(1-x)^2}$$

$$= \frac{\frac{x^2-x}{\left(1-x^2\right)^{1/2}} + \frac{\left(1-x^2\right)}{\left(1-x^2\right)^{1/2}}}{(1-x)^2}$$

$$= \frac{1-x}{\left(1-x^2\right)^{1/2}(1-x)^2}$$

$$= \frac{1}{(1-x)\sqrt{1-x^2}}$$

91. $y=\left(\frac{x^2-x-1}{x^2+1}\right)^3$

Using the Extended Power Rule and the Quotient Rule, we have

$$\frac{dy}{dx}$$
$$=3\left(\frac{x^2-x-1}{x^2+1}\right)^2\left[\frac{(x^2+1)(2x-1)-(x^2-x-1)(2x)}{(x^2+1)^2}\right]$$
$$=3\left(\frac{x^2-x-1}{x^2+1}\right)^2\left[\frac{x^2+4x-1}{(x^2+1)^2}\right]$$
$$=\frac{3(x^2-x-1)^2(x^2+4x-1)}{(x^2+1)^4}$$

92. $g(x)=\sqrt{\frac{x^2-4x}{2x+1}}=\left(\frac{x^2-4x}{2x+1}\right)^{1/2}$

$$g'(x)$$
$$=\frac{1}{2}\left(\frac{x^2-4x}{2x+1}\right)^{-1/2}\left[\frac{(2x+1)(2x-4)-(x^2-4x)(2)}{(2x+1)^2}\right]$$
$$=\frac{1}{2}\left(\frac{2x+1}{x^2-4x}\right)^{1/2}\left[\frac{4x^2-6x-4-2x^2+8x}{(2x+1)^2}\right]$$
$$=\frac{1}{2}\left(\frac{2x+1}{x^2-4x}\right)^{1/2}\left[\frac{2x^2+2x-4}{(2x+1)^2}\right]$$
$$=\frac{1}{2}\left(\frac{2x+1}{x^2-4x}\right)^{1/2}\left[\frac{2(x^2+x-2)}{(2x+1)^2}\right]$$
$$=\frac{x^2+x-2}{(2x+1)^{3/2}(x^2-4x)^{1/2}}$$

93. $f(t)=\sqrt{3t+\sqrt{t}}=\left(3t+t^{1/2}\right)^{1/2}$

Using the Extended Power Rule, we have

$$f'(t)=\frac{1}{2}\left(3t+t^{1/2}\right)^{-1/2}\left(3+\frac{1}{2}t^{-1/2}\right)$$
$$=\frac{3+\frac{1}{2\sqrt{t}}}{2\sqrt{3t+\sqrt{t}}}$$
$$=\frac{\frac{3}{1}\cdot\frac{2\sqrt{t}}{2\sqrt{t}}+\frac{1}{2\sqrt{t}}}{2\sqrt{3t+\sqrt{t}}}$$

Simplifying, we have:

$$f'(t)=\frac{\frac{6\sqrt{t}+1}{2\sqrt{t}}}{2\sqrt{3t+\sqrt{t}}}$$
$$=\frac{6\sqrt{t}+1}{4\sqrt{t}\sqrt{3t+\sqrt{t}}}$$

94. $F(x)=\left[6x(3-x)^5+2\right]^4$

$$F'(x)$$
$$=4\left(6x(3-x)^5+2\right)^3\left[6x\left(5(3-x)^4(-1)\right)+(3-x)^5(6)\right]$$
$$=4\left(6x(3-x)^5+2\right)^3\left[-30x(3-x)^4+6(3-x)^5\right]$$
$$=4\left(6x(3-x)^5+2\right)^3\left[6(3-x)^4\left(-5x+(3-x)\right)\right]$$
$$=24(3-x)^4(3-6x)\left(6x(3-x)^5+2\right)^3$$
$$=-72(3-x)^4(2x-1)\left(6x(3-x)^5+2\right)^3$$

95. a) Applying the product rule, we have:

$$\frac{d}{dx}\left[f(x)\right]^3$$
$$=\frac{d}{dx}\left[\left[f(x)\right]^2\cdot\left[f(x)\right]\right]$$
$$=\left[f(x)\right]^2\frac{d}{dx}\left(f(x)\right)+\frac{d}{dx}\left[f(x)\right]^2 f(x)$$
$$=\left[f(x)\right]^2 f'(x)+\frac{d}{dx}\left[f(x)\right]^2 f(x)$$

We apply the product rule to $\frac{d}{dx}\left[f(x)\right]^2$

$$\frac{d}{dx}\left[f(x)\right]^2=f(x)f'(x)+f'(x)f(x)$$
$$=2f(x)f'(x).$$

Therefore, we now have:

$$\frac{d}{dx}\left[f(x)\right]^3=\left[f(x)\right]^2 f'(x)+2\left[f(x)\right]^2 f'(x)$$
$$=3\left[f(x)\right]^2 f'(x)$$

b) Applying the product rule we have:

$$\frac{d}{dx}\left[f(x)\right]^4$$
$$=\frac{d}{dx}\left[\left[f(x)\right]^3\cdot\left[f(x)\right]\right]$$
$$=\left[f(x)\right]^3\frac{d}{dx}\left(f(x)\right)+\frac{d}{dx}\left[f(x)\right]^3 f(x)$$
$$=\left[f(x)\right]^3 f'(x)+\frac{d}{dx}\left[f(x)\right]^3 f(x).$$

From part (a) we know that:

$$\frac{d}{dx}\left[f(x)\right]^3=3\left[f(x)\right]^2 f'(x)$$

Therefore, substituting into the derivative we have:

$$\frac{d}{dx}\left[f(x)\right]^4$$
$$=\left[f(x)\right]^3 f'(x)+3\left[f(x)\right]^2 f'(x)f(x)$$
$$=4\left[f(x)\right]^3 f'(x)$$

96. Let $Q(x)=\dfrac{N(x)}{D(x)}$.

Then. $Q(x)=N(x)\left[D(x)\right]^{-1}$

Therefore,

$$Q'(x)=N(x)\frac{d}{dx}\left[D(x)\right]^{-1}+\left[D(x)\right]^{-1}\frac{d}{dx}N(x)$$

Product Rule

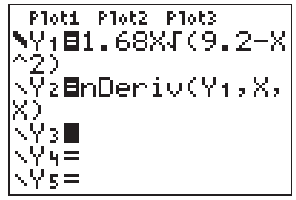
$$=N(x)(-1)\left[D(x)\right]^{-2}\cdot D'(x)+\left[D(x)\right]^{-1}N'(x)$$

Extended Power Rule

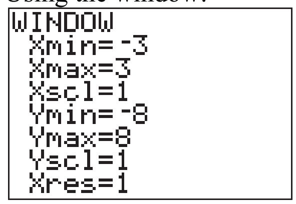
$$=\frac{-N(x)\cdot D'(x)}{\left[D(x)\right]^2}+\frac{N'(x)}{D(x)}$$
$$=\frac{-N(x)\cdot D'(x)}{\left[D(x)\right]^2}+\frac{N'(x)\cdot D(x)}{\left[D(x)\right]^2}$$
$$=\frac{D(x)\cdot N'(x)-N(x)\cdot D'(x)}{\left[D(x)\right]^2}$$

97. $f(x)=1.68x\sqrt{9.2-x^2}$; $[-3,3]$

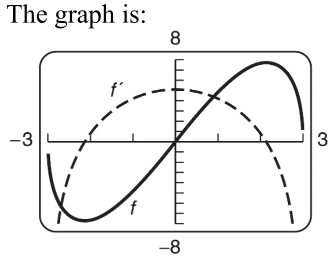
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative at the top of the next column.



Using the window:

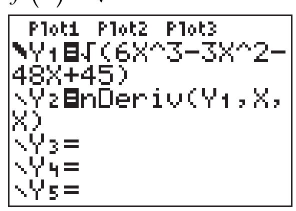


The graph is:

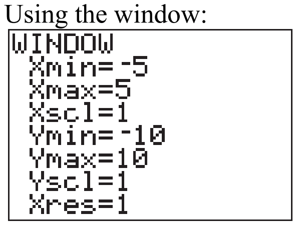


Note, the function $f(x)$ is the solid graph. The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency. We estimate the points at which the tangent lines are horizontal are $(-2.14476,-7.728)$ and $(2.14476,7.728)$.

98. $f(x)=\sqrt{6x^3-3x^2-48x+45}$, $[-5,5]$

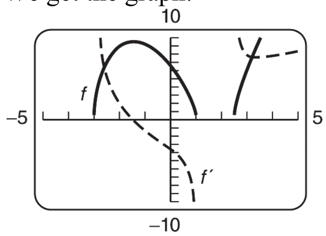


Using the window:



The graph is displayed on the next page.

We get the graph:



Note, the function $f(x)$ is the solid graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency. We estimate the point at which the tangent line is horizontal to be $(-1.47481, 9.4878)$.

99. $f(x) = x\sqrt{4-x^2} = x\left(4-x^2\right)^{1/2}$

$$f'(x) = x\left[\frac{1}{2}\left(4-x^2\right)^{-1/2}(-2x)\right] + \left(4-x^2\right)^{1/2}(1)$$

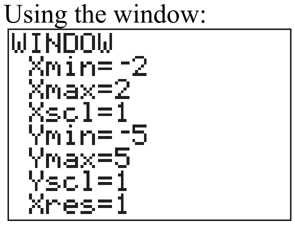
$$= \frac{-x^2}{\left(4-x^2\right)^{1/2}} + \left(4-x^2\right)^{1/2}$$

$$= \frac{-x^2}{\left(4-x^2\right)^{1/2}} + \frac{\left(4-x^2\right)}{\left(4-x^2\right)^{1/2}}$$

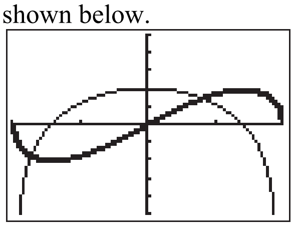
$$= \frac{4-2x^2}{\left(4-x^2\right)^{1/2}}$$

$$= \frac{4-2x^2}{\sqrt{4-x^2}}$$

Using the window:



The graph of the function and the derivative are shown below.



Note: The graph of the function is the thicker graph.

100. $f(x) = \left(\sqrt{2x-1} + x^3\right)^5 = \left(\left(2x-1\right)^{1/2} + x^3\right)^5$

$$f'(x) = 5\left(\left(2x-1\right)^{1/2} + x^3\right)^4\left[\frac{1}{2}\left(2x-1\right)^{-1/2}(2) + 3x^2\right]$$

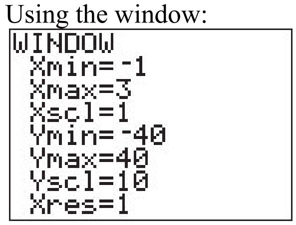
$$= 5\left(\left(2x-1\right)^{1/2} + x^3\right)^4\left[\frac{1}{\left(2x-1\right)^{1/2}} + 3x^2\right]$$

Simplifying, we have:

$$f'(x) = 5\left(\left(2x-1\right)^{1/2} + x^3\right)^4\left[\frac{3x^2\left(2x-1\right)^{1/2} + 1}{\left(2x-1\right)^{1/2}}\right]$$

$$= \frac{5\left(\sqrt{2x-1} + x^3\right)^4\left(3x^2\sqrt{2x-1} + 1\right)}{\sqrt{2x-1}}$$

Using the window:

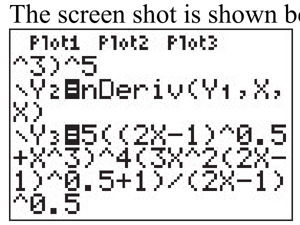


First we enter the function $Y_1 = \left(\left(2x-1\right)^{0.5} + x^3\right)^5$ into the graphing editor.

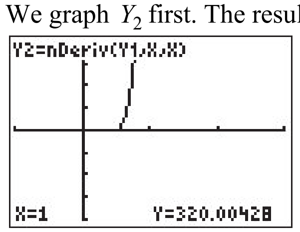
Next, we enter $Y_2 = nDeriv\left(y_1, x, x\right)$ into the graphing editor. Finally, we enter

$$Y_3 = \frac{5\left(\left(2x-1\right)^{0.5} + x^3\right)^4\left(3x^2\left(2x-1\right)^{0.5} + 1\right)}{\left(2x-1\right)^{0.5}}.$$

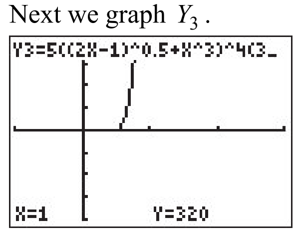
The screen shot is shown below:



We graph Y_2 first. The resulting graph is shown:



Next we graph Y_3.



The two graphs coincide verifying the result.

Exercise Set 1.8

1. $y = x^4 - 7$

$\frac{dy}{dx} = 4x^{4-1} = 4x^3$ First Derivative

$\frac{d^2y}{dx^2} = 4\left(3x^{3-1}\right) = 12x^2$ Second Derivative

2. $y = x^5 + 9$

$\frac{dy}{dx} = 5x^4$

$\frac{d^2y}{dx^2} = 20x^3$

3. $y = 2x^4 - 5x$

$\frac{dy}{dx} = 2\left(4x^3\right) - 5 =$

$= 8x^3 - 5$ First Derivative

$\frac{d^2y}{dx^2} = 8\left(3x^2\right)$

$= 24x^2$ Second Derivative

4. $y = 5x^3 + 4x$

$\frac{dy}{dx} = 15x^2 + 4$

$\frac{d^2y}{dx^2} = 30x$

5. $y = 4x^2 - 5x + 7$

$\frac{dy}{dx} = 4\left(2x^{2-1}\right) - 5$

$= 8x - 5$ First Derivative

$\frac{d^2y}{dx^2} = 8$ Second Derivative

6. $y = 4x^2 + 3x - 1$

$\frac{dy}{dx} = 8x + 3$

$\frac{d^2y}{dx^2} = 8$

7. $y = 7x + 2$

$\frac{dy}{dx} = 7$ First Derivative

$\frac{d^2y}{dx^2} = 0$ Second Derivative

8. $y = 6x - 3$

$\frac{dy}{dx} = 6$

$\frac{d^2y}{dx^2} = 0$

9. $y = \frac{1}{x^3} = x^{-3}$

$\frac{dy}{dx} = -3x^{-3-1}$

$= -3x^{-4}$

$= \frac{-3}{x^4}$ First Derivative

$\frac{d^2y}{dx^2} = -3\left(-4x^{-4-1}\right)$

$= 12x^{-5}$

$= \frac{12}{x^5}$ Second Derivative

10. $y = \frac{1}{x^2} = x^{-2}$

$\frac{dy}{dx} = -2x^{-3} = \frac{-2}{x^3}$

$\frac{d^2y}{dx^2} = 6x^{-4} = \frac{6}{x^4}$

11. $y = \sqrt{x} = x^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}x^{1/2-1}$

$= \frac{1}{2}x^{-1/2}$

$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$ First Derivative

$\frac{d^2y}{dx^2} = \frac{1}{2}\cdot\left(-\frac{1}{2}x^{-1/2-1}\right)$

$= -\frac{1}{4}x^{-3/2}$

$= -\frac{1}{4x^{3/2}} = -\frac{1}{4\sqrt{x^3}}$ Second Derivative

12. $y = \sqrt[4]{x} = x^{1/4}$

$$\frac{dy}{dx} = \frac{1}{4}x^{1/4-1} = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}} = \frac{1}{4\cdot\sqrt[4]{x^3}}$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}\left(-\frac{3}{4}\right)x^{-3/4-1}$$

$$= -\frac{3}{16}x^{-7/4}$$

$$= -\frac{3}{16x^{7/4}} = -\frac{3}{16x\sqrt[4]{3}}$$

13. $f(x) = x^3 - \frac{5}{x} = x^3 - 5x^{-1}$

$$f'(x) = 3x^{3-1} + 5\left(-1x^{-1-1}\right)$$

$$= 3x^2 + 5x^{-2}$$

$$= 3x^2 + \frac{5}{x^2} \quad \text{First Derivative}$$

$$f''(x) = 3\left(2x^{2-1}\right) + 5\left(-2x^{-2-1}\right)$$

$$= 6x - 10x^{-3}$$

$$= 6x - \frac{10}{x^3} \quad \text{Second Derivative}$$

14. $f(x) = x^4 + \frac{3}{x}$

$$f'(x) = 4x^3 - 3x^{-2}$$

$$= 4x^3 - \frac{3}{x^2}$$

$$f''(x) = 12x^2 + 6x^{-3}$$

$$= 12x^2 + \frac{6}{x^3}$$

15. $f(x) = x^{1/5}$

$$f'(x) = \frac{1}{5}x^{1/5-1}$$

$$= \frac{1}{5}x^{-4/5}$$

$$= \frac{1}{5x^{4/5}} \quad \text{First Derivative}$$

$$f''(x) = \frac{1}{5}\left(-\frac{4}{5}\right)x^{-4/5-1}$$

$$= -\frac{4}{25}x^{-9/5}$$

$$= -\frac{4}{25x^{9/5}} \quad \text{Second Derivative}$$

16. $f(x) = x^{1/3}$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$f''(x) = \frac{1}{3}\left(-\frac{2}{3}\right)x^{-5/3} = -\frac{2}{9x^{5/3}}$$

17. $f(x) = 2x^{-2}$

$$f'(x) = 2\left(-2x^{-2-1}\right)$$

$$= -4x^{-3}$$

$$= -\frac{4}{x^3} \quad \text{First Derivative}$$

$$f''(x) = -4\left(-3x^{-3-1}\right)$$

$$= 12x^{-4}$$

$$= \frac{12}{x^4} \quad \text{Second Derivative}$$

18. $f(x) = 4x^{-3}$

$$f'(x) = -12x^{-4} = -\frac{12}{x^4}$$

$$f''(x) = 48x^{-5} = \frac{48}{x^5}$$

19. $f(x) = \left(x^2 + 3x\right)^7$

$$f'(x) = 7\left(x^2+3x\right)^{7-1}(2x+3) \quad \text{Theorem 7}$$

$$= 7(2x+3)\left(x^2+3x\right)^6 \quad \text{First Derivative}$$

$$f''(x) = 7(2x+3)\left(6\left(x^2+3x\right)^{6-1}(2x+3)\right) +$$

$$7\left(x^2+6x\right)^6(2) \quad \text{Theorem 5}$$

$$= 42(2x+3)^2\left(x^2+3x\right)^5 + 14\left(x^2+3x\right)^6$$

We can simplify the second derivative by factoring out common factors.

$$f''(x)$$

$$= 14\left(x^2+3x\right)^5\left[3(2x+3)^2 + \left(x^2+3x\right)\right]$$

$$= 14\left(x^2+3x\right)^5\left[3\left(4x^2+12x+9\right) + \left(x^2+3x\right)\right]$$

$$= 14\left(x^2+3x\right)^5\left[12x^2+36x+27+x^2+3x\right]$$

$$= 14\left(x^2+3x\right)^5\left(13x^2+39x+27\right) \quad \text{Second Derivative}$$

20. $f(x)=\left(x^3+2x\right)^6$

$f'(x)=6\left(x^3+2x\right)^5\left(3x^2+2\right)$

$f''(x)=6\left[\left(x^3+2x\right)^5(6x)\right]+$

$6\left[\left(3x^2+2\right)\cdot 5\left(x^3+2x\right)^4\left(3x^2+2\right)\right]$

$=36x\left(x^3+2x\right)^5+30\left(3x^2+2\right)^2\left(x^3+2x\right)^4$

$=6\left(x^3+2x\right)^4\left[6x\left(x^3+2x\right)+5\left(3x^2+2\right)^2\right]$

$=6\left(x^3+2x\right)^4\left[6x^4+12x^2+45x^4+60x^2+20\right]$

$=6\left(x^3+2x\right)^4\left(51x^4+72x^2+20\right)$

21. $f(x)=\left(3x^2+2x+1\right)^5$

$f'(x)=5\left(3x^2+2x+1\right)^{5-1}(6x+2)$ Theorem 7

$=5\left(3x^2+2x+1\right)^4(6x+2)$ First Derivative

$f''(x)=5\left(3x^2+2x+1\right)^4(6)+$ Theorem 5

$5(6x+2)\cdot 4\left(3x^2+2x+1\right)^3(6x+2)$

$=30\left(3x^2+2x+1\right)^4+$

$20(6x+2)^2\left(3x^2+2x+1\right)^3$

$=10\left(3x^2+2x+1\right)^3\left[3\left(3x^2+2x+1\right)+2\left(36x^2+24x+4\right)\right]$

$=10\left(3x^2+2x+1\right)^3\left[9x^2+6x+3+72x^2+48x+8\right]$

$=10\left(3x^2+2x+1\right)^3\left(81x^2+54x+11\right)$

Second Derivative

22. $f(x)=\left(2x^2-3x+1\right)^{10}$

$f'(x)=10\left(2x^2-3x+1\right)^9(4x-3)$

$f''(x)=10\left(2x^2-3x+1\right)^9(4)+$

$10(4x-3)\cdot 9\left(2x^2-3x+1\right)^8(4x-3)$

$=40\left(2x^2-3x+1\right)^9+$

$90(4x-3)^2\left(2x^2-3x+1\right)^8$

$=10\left(2x^2-3x+1\right)^8\left(4\left(2x^2-3x+1\right)+9\left(16x^2-24x+9\right)\right)$

$=10\left(2x^2-3x+1\right)^8\left(8x^2-12x+4+144x^2-216x+81\right)$

$=10\left(2x^2-3x+1\right)^8\left(152x^2-228x+85\right)$

23. $f(x)=\sqrt[4]{\left(x^2+1\right)^3}=\left(x^2+1\right)^{3/4}$

$f'(x)=\frac{3}{4}\left(x^2+1\right)^{-1/4}(2x)$ Theorem 7

$=\frac{3}{2}x\left(x^2+1\right)^{-1/4}$ First Derivative

$f''(x)=\frac{3}{2}x\cdot\frac{-1}{4}\left(x^2+1\right)^{-5/4}(2x)+$

$\frac{3}{2}\left(x^2+1\right)^{-1/4}(1)$ Theorem 5

$=-\frac{3}{4}x^2\left(x^2+1\right)^{-5/4}+\frac{3}{2}\left(x^2+1\right)^{-1/4}$

$=\frac{-3x^2}{4\left(x^2+1\right)^{5/4}}+\frac{3}{2\left(x^2+1\right)^{1/4}}$

We can simplify the second derivative by finding a common denominator and combining the fractions.

$f''(x)=\frac{-3x^2}{4\left(x^2+1\right)^{5/4}}+\frac{3}{2\left(x^2+1\right)^{1/4}}\cdot\frac{2\left(x^2+1\right)}{2\left(x^2+1\right)}$

$=\frac{-3x^2}{4\left(x^2+1\right)^{5/4}}+\frac{6\left(x^2+1\right)}{4\left(x^2+1\right)^{5/4}}$

$=\frac{3x^2+6}{4\left(x^2+1\right)^{5/4}}$

$=\frac{3\left(x^2+2\right)}{4\left(x^2+1\right)^{5/4}}$

24. $f(x)=\sqrt[3]{\left(x^2-1\right)^2}=\left(x^2-1\right)^{2/3}$

$f'(x)=\frac{2}{3}\left(x^2-1\right)^{-1/3}(2x)$

$=\frac{4}{3}x\left(x^2-1\right)^{-1/3}$

$f''(x)=\frac{4}{3}x\cdot\frac{-1}{3}\left(x^2-1\right)^{-4/3}(2x)$

$+\frac{4}{3}\left(x^2-1\right)^{-1/3}(1)$

$=\frac{-8}{9}x^2\left(x^2-1\right)^{-4/3}+\frac{4}{3}\left(x^2-1\right)^{-1/3}$

$=\frac{-8x^2}{9\left(x^2-1\right)^{4/3}}+\frac{4}{3\left(x^2-1\right)^{1/3}}$

$=\frac{-8x^2}{9\left(x^2-1\right)^{4/3}}+\frac{12\left(x^2-1\right)}{9\left(x^2-1\right)^{4/3}}$

$=\frac{4x^2-12}{9\left(x^2-1\right)^{4/3}}$

$=\frac{4\left(x^2-3\right)}{9\left(x^2-1\right)^{4/3}}$

25. $y=x^{3/2}-5x$

$y'=\frac{3}{2}x^{3/2-1}-5$

$=\frac{3}{2}x^{1/2}-5$ First Derivative

$y''=\frac{3}{2}\left(\frac{1}{2}x^{1/2-1}\right)$

$=\frac{3}{4}x^{-1/2}$

$=\frac{3}{4\sqrt{x}}$ Second Derivative

26. $y=x^{2/3}+4x$

$y'=\frac{2}{3}x^{-1/3}+4=\frac{2}{3x^{1/3}}+4$

$y''=-\frac{2}{9}x^{-4/3}=-\frac{2}{9x^{4/3}}$

27. $y=\left(x^3-x\right)^{3/4}$

$y'=\frac{3}{4}\left(x^3-x\right)^{3/4-1}\left(3x^2-1\right)$ Theorem 7

$=\frac{3}{4}\left(x^3-x\right)^{-1/4}\left(3x^2-1\right)$ First Derivative

$y''=\frac{3}{4}\left(x^3-x\right)^{-1/4}(6x)+$ Theorem 5

$\frac{3}{4}\left(3x^2-1\right)\cdot\frac{-1}{4}\left(x^3-x\right)^{-1/4-1}\left(3x^2-1\right)$

$=\frac{9}{2}x\left(x^3-x\right)^{-1/4}+$

$\frac{-3}{16}\left(3x^2-1\right)^2\left(x^3-x\right)^{-5/4}$

$=\frac{9x}{2\left(x^3-x\right)^{1/4}}-\frac{3\left(3x^2-1\right)^2}{16\left(x^3-x\right)^{5/4}}$

The second derivative can be simplified by finding a common denominator and combining the fractions.

$y''=\frac{9x}{2\left(x^3-x\right)^{1/4}}\cdot\frac{8\left(x^3-x\right)}{8\left(x^3-x\right)}-\frac{3\left(3x^2-1\right)^2}{16\left(x^3-x\right)^{5/4}}$

$=\frac{72x^4-72x^2}{16\left(x^3-x\right)^{5/4}}-\frac{3\left(9x^4-6x^2+1\right)}{16\left(x^3-x\right)^{5/4}}$

$=\frac{72x^4-72x^2-27x^4+18x^2-3}{16\left(x^3-x\right)^{5/4}}$

$=\frac{45x^4-54x^2-3}{16\left(x^3-x\right)^{5/4}}$ Second Derivative

28. $y=\left(x^4+x\right)^{2/3}$

$y'=\frac{2}{3}\left(x^4+x\right)^{-1/3}\left(4x^3+1\right)$

The solution is continued on the next page.

Differentiating the derivative on the previous page, we find the second derivative.

$$y'' = \frac{2}{3}\left[\left(x^4+x\right)^{-1/3}\left(12x^2\right)+\left(4x^3+1\right)\left(-\frac{1}{3}\left(x^4+x\right)^{-4/3}\left(4x^3+1\right)\right)\right]$$

$$= \frac{2}{3}\left[\frac{12x^2}{\left(x^4+x\right)^{1/3}} - \frac{\left(4x^3+1\right)^2}{3\left(x^4+x\right)^{4/3}}\right]$$

$$= \frac{2}{3}\left[\frac{36x^2\left(x^4+x\right)}{3\left(x^4+x\right)^{4/3}} - \frac{16x^6+8x^3+1}{3\left(x^4+x\right)^{4/3}}\right]$$

$$= \frac{2}{3}\left[\frac{20x^6+28x^3-1}{3\left(x^4+x\right)^{4/3}}\right]$$

$$= \frac{40x^6+56x^3-2}{9\left(x^4+x\right)^{4/3}}$$

29. $y = 3x^{4/3} - x^{1/2}$

$$y' = 3\cdot\frac{4}{3}x^{4/3-1} - \frac{1}{2}x^{1/2-1}$$

$$= 4x^{1/3} - \frac{1}{2}x^{-1/2} \qquad \text{First Derivative}$$

$$y'' = 4\cdot\frac{1}{3}x^{1/3-1} - \frac{1}{2}\cdot\frac{-1}{2}x^{-1/2-1}$$

$$= \frac{4}{3}x^{-2/3} + \frac{1}{4}x^{-3/2}$$

$$= \frac{4}{3x^{2/3}} + \frac{1}{4x^{3/2}} \qquad \text{Second Derivative}$$

30. $y = 2x^{5/4} + x^{1/2}$

$$y' = 2\cdot\frac{5}{4}x^{5/4-1} + \frac{1}{2}x^{1/2-1}$$

$$= \frac{5}{2}x^{1/4} + \frac{1}{2}x^{-1/2}$$

$$y'' = \frac{5}{2}\cdot\frac{1}{4}x^{1/4-1} + \frac{1}{2}\cdot\frac{-1}{2}x^{-1/2-1}$$

$$= \frac{5}{8}x^{-3/4} - \frac{1}{4}x^{-3/2}$$

$$= \frac{5}{8x^{3/4}} - \frac{1}{4x^{3/2}}$$

31. $y = \frac{2}{x^3} + \frac{1}{x^2} = 2x^{-3} + x^{-2}$

$$y' = 2\left(-3x^{-3-1}\right) + \left(-2x^{-2-1}\right)$$

$$= -6x^{-4} - 2x^{-3} \qquad \text{First Derivative}$$

$$y'' = -6\left(-4x^{-4-1}\right) - 2\left(-3x^{-3-1}\right)$$

$$= 24x^{-5} + 6x^{-4}$$

$$= \frac{24}{x^5} + \frac{6}{x^4} \qquad \text{Second Derivative}$$

32. $y = \frac{3}{x^4} - \frac{1}{x} = 3x^{-4} - x^{-1}$

$$y' = -12x^{-5} + x^{-2}$$

$$y'' = 60x^{-6} - 2x^{-3} = \frac{60}{x^6} - \frac{2}{x^3}$$

33. $y = \left(x^3-2\right)\left(5x+1\right)$

$$y' = \left(x^3-2\right)\left(5\right) + \left(5x+1\right)\left(3x^2\right) \qquad \text{Theorem 5}$$

$$= 5x^3 - 10 + 15x^3 + 3x^2$$

$$= 20x^3 + 3x^2 - 10 \qquad \text{First Derivative}$$

$$y'' = 20\left(3x^{3-1}\right) + 3\left(2x^{2-1}\right)$$

$$= 60x^2 + 6x \qquad \text{Second Derivative}$$

34. $y = \left(x^2+3\right)\left(4x-1\right)$

$$y' = \left(x^2+3\right)\left(4\right) + \left(4x-1\right)\left(2x\right)$$

$$= 4x^2 + 12 + 8x^2 - 2x$$

$$= 12x^2 - 2x + 12$$

$$y'' = 24x - 2$$

35. $y = \frac{3x+1}{2x-3}$

$$y' = \frac{(2x-3)(3)-(3x+1)(2)}{(2x-3)^2} \qquad \text{Theorem 6}$$

$$= \frac{6x-9-6x-2}{(2x-3)^2}$$

$$= \frac{-11}{(2x-3)^2} \qquad \text{First Derivative}$$

The solution is continued on the next page.

Differentiating the derivative on the previous page, we find the second derivative.

$$y'' = \frac{(2x-3)^2(0)-(-11)\left(2(2x-3)^{2-1}(2)\right)}{\left((2x-3)^2\right)^2}$$

Theorem 6 and Theorem 7

$$= \frac{44(2x-3)}{(2x-3)^4}$$

$$= \frac{44}{(2x-3)^3} \quad \text{Second Derivative}$$

36. $y = \dfrac{2x+3}{5x-1}$

$$y' = \frac{(5x-1)(2)-(2x+3)(5)}{(5x-1)^2}$$

$$= \frac{-17}{(5x-1)^2} = -17(5x-1)^{-2}$$

$$y'' = -17\left(-2(5x-1)^{-3}(5)\right) \quad \text{Extended Power Rule}$$

$$= 170(5x-1)^{-3}$$

$$= \frac{170}{(5x-1)^3}$$

37. $y = x^5$

$$\frac{dy}{dx} = 5x^{5-1} = 5x^4 \quad \text{First Derivative}$$

$$\frac{d^2y}{dx^2} = 5\left(4x^{4-1}\right) = 20x^3 \quad \text{Second Derivative}$$

$$\frac{d^3y}{dx^3} = 20\left(3x^{3-1}\right) = 60x^2 \quad \text{Third Derivative}$$

$$\frac{d^4y}{dx^4} = 60\left(2x^{2-1}\right) = 120x \quad \text{Fourth Derivative}$$

38. $y = x^4$

$$\frac{dy}{dx} = 4x^3$$

$$\frac{d^2y}{dx^2} = 12x^2$$

$$\frac{d^3y}{dx^3} = 24x$$

$$\frac{d^4y}{dx^4} = 24$$

39. $y = x^6 - x^3 + 2x$

$$\frac{dy}{dx} = 6x^{6-1} - 3x^{3-1} + 2$$

$$= 6x^5 - 3x^2 + 2 \quad \text{First Derivative}$$

$$\frac{d^2y}{dx^2} = 6\left(5x^{5-1}\right) - 3\left(2x^{2-1}\right)$$

$$= 30x^4 - 6x \quad \text{Second Derivative}$$

$$\frac{d^3y}{dx^3} = 30\left(4x^{4-1}\right) - 6$$

$$= 120x^3 - 6 \quad \text{Third Derivative}$$

$$\frac{d^4y}{dx^4} = 120\left(3x^{3-1}\right)$$

$$= 360x^2 \quad \text{Fourth Derivative}$$

$$\frac{d^5y}{dx^5} = 360\left(2x^{2-1}\right)$$

$$= 720x \quad \text{Fifth Derivative}$$

40. $y = x^7 - 8x^2 + 2$

$$\frac{dy}{dx} = 7x^6 - 16x$$

$$\frac{d^2y}{dx^2} = 42x^5 - 16$$

$$\frac{d^3y}{dx^3} = 210x^4$$

$$\frac{d^4y}{dx^4} = 840x^3$$

$$\frac{d^5y}{dx^5} = 2520x^2$$

$$\frac{d^6y}{dx^6} = 5040x$$

41. $f(x) = x^{-3} + 2x^{1/3}$

$$f'(x) = -3x^{-3-1} + 2\left(\frac{1}{3}x^{1/3-1}\right)$$

$$= -3x^{-4} + \frac{2}{3}x^{-2/3} \quad \text{First Derivative}$$

$$f''(x) = -3\left(-4x^{-4-1}\right) + \frac{2}{3}\cdot\frac{-2}{3}x^{-2/3-1}$$

$$= 12x^{-5} - \frac{4}{9}x^{-5/3} \quad \text{Second Derivative}$$

The solution is continued on the next page.

Continuing from the previous page:

$$f'''(x) = 12\left(-5x^{-5-1}\right) - \frac{4}{9}\cdot\frac{-5}{3}x^{-5/3-1}$$

$$= -60x^{-6} + \frac{20}{27}x^{-8/3} \quad \text{Third Derivative}$$

$$f^{(4)}(x) = -60\left(-6x^{-6-1}\right) + \frac{20}{27}\cdot\frac{-8}{3}x^{-8/3-1}$$

$$= 360x^{-7} - \frac{160}{81}x^{-11/3} \quad \text{Fourth Derivative}$$

$$f^{(5)}(x) = 360\left(-7x^{-7-1}\right) - \frac{160}{81}\cdot\frac{-11}{3}x^{-11/3-1}$$

$$= -2520x^{-8} + \frac{1760}{243}x^{-14/3} \quad \text{Fifth Derivative}$$

42. $f(x) = x^{-2} - x^{1/2}$

$$f'(x) = -2x^{-2-1} - \frac{1}{2}x^{1/2-1}$$

$$= -2x^{-3} - \frac{1}{2}x^{-1/2}$$

$$f''(x) = -2\left(-3x^{-3-1}\right) - \frac{1}{2}\cdot\frac{-1}{2}x^{-1/2-1}$$

$$= 6x^{-4} + \frac{1}{4}x^{-3/2}$$

$$f'''(x) = 6\left(-4x^{-4-1}\right) + \frac{1}{4}\cdot\frac{-3}{2}x^{-3/2-1}$$

$$= -24x^{-5} - \frac{3}{8}x^{-5/2}$$

$$f^{(4)}(x) = -24\left(-5x^{-5-1}\right) - \frac{3}{8}\cdot\frac{-5}{2}x^{-5/2-1}$$

$$= 120x^{-6} + \frac{15}{16}x^{-7/2}$$

43. $g(x) = x^4 - 3x^3 - 7x^2 - 6x + 9$

$$g'(x) = 4x^{4-1} - 3\left(3x^{3-1}\right) - 7\left(2x^{2-1}\right) - 6$$

$$= 4x^3 - 9x^2 - 14x - 6 \quad \text{First Derivative}$$

$$g''(x) = 4\left(3x^{3-1}\right) - 9\left(2x^{2-1}\right) - 14$$

$$= 12x^2 - 18x - 14 \quad \text{Second Derivative}$$

$$g'''(x) = 12\left(2x^{2-1}\right) - 18$$

$$= 24x - 18 \quad \text{Third Derivative}$$

$$g^{(4)}(x) = 24 \quad \text{Fourth Derivative}$$

$$g^{(5)}(x) = 0 \quad \text{Fifth Derivative}$$

$$g^{(6)}(x) = 0 \quad \text{Sixth Derivative}$$

44. $g(x) = 6x^5 + 2x^4 - 4x^3 + 7x^2 - 8x + 3$

$$g'(x) = 30x^4 + 8x^3 - 12x^2 + 14x - 8$$

$$g''(x) = 120x^3 + 24x^2 - 24x + 14$$

$$g'''(x) = 360x^2 + 48x - 24$$

$$g^{(4)}(x) = 720x + 48$$

$$g^{(5)}(x) = 720$$

$$g^{(6)}(x) = 0$$

$$g^{(7)}(x) = 0$$

45. $s(t) = -10t^2 + 2t + 5$

a) $v(t) = s'(t) = -20t + 2$

b) $a(t) = v'(t) = s''(t) = -20$

c) When $t = 1$,

$v(1) = -20(1) + 2 = -18\,\frac{\text{m}}{\text{sec}}$

$a(1) = -20\,\frac{\text{m}}{\text{sec}^2}$

After 1 second, the velocity is -18 meters per second, and the acceleration is -20 meters per second squared.

46. $s(t) = t^3 + t$

a) $v(t) = s'(t) = 3t^2 + 1$

b) $a(t) = v'(t) = s''(t) = 6t$

c) When $t = 4$

$v(4) = 3(4)^2 + 1 = 49$

$a(4) = 6(4) = 24$

After 4 seconds, the velocity is 49 feet per second, and the acceleration is 24 feet per second squared.

47. $s(t) = 3t + 10$

a) $v(t) = s'(t) = 3$

b) $a(t) = v'(t) = s''(t) = 0$

c) When $t = 2$

$v(2) = 3$

$a(2) = 0$

After 2 hours, the velocity is 3 miles per hour, and the acceleration is 0 miles per hour squared.

d) ✎ Answers will vary. Uniform motion means that the object is moving with a constant velocity. Since the object's velocity is not changing, the object's acceleration will be zero.

48. $s(t) = t^2 - \frac{1}{2}t + 3$

a) $v(t) = s'(t) = 2t - \frac{1}{2}$

b) $a(t) = v'(t) = s''(t) = 2$

c) When $t = 1$,

$v(1) = 2(1) - \frac{1}{2} = \frac{3}{2} = 1.5 \frac{\text{m}}{\text{sec}}$

$a(1) = 2 \frac{\text{m}}{\text{sec}^2}$

49. $s(t) = 16t^2$

a) When $t = 3$, $s(3) = 16(3)^2 = 144$.
The hammer falls 144 feet in 3 seconds.

b) $v(t) = s'(t) = 32t$

When $t = 3$, $v(3) = 32(3) = 96$

the hammer is falling at 96 feet per second after 3 seconds.

c) $a(t) = v'(t) = s''(t) = 32$

When $t = 3$, $a(3) = 32$

the hammer is accelerating at 32 feet per second squared after 3 seconds.

50. $s(t) = 16t^2$

a) When $t = 2$, $s(2) = 16(2)^2 = 64$.
The bolt falls 64 feet in 2 seconds.

b) $v(t) = s'(t) = 32t$

When $t = 2$, $v(2) = 32(2) = 64$

the bolt is falling at 64 feet per second after 2 seconds.

c) $a(t) = v'(t) = s''(t) = 32$

When $t = 2$, $a(2) = 32$

the bolt is accelerating at 32 feet per second squared after 2 seconds.

51. $s(t) = 4.905t^2$

The velocity and acceleration are given by:

$v(t) = s'(t) = 9.81t$

$a(t) = v'(t) = s''(t) = 9.81$

After 2 seconds, we have

$v(2) = 9.81(2) = 19.62$

The stone is falling at 19.62 meters per second.

$a(2) = 9.81$

The stone is accelerating at 9.81 meters per second squared.

52. $s(t) = 4.905t^2$

$v(t) = s'(t) = 9.81t$

$a(t) = v'(t) = s''(t) = 9.81$

After 3 seconds, we have

$v(3) = 9.81(3) = 29.43$

The stone is falling at 29.43 meters per second.

$a(3) = 9.81$

The stone is accelerating at 9.81 meters per second squared.

53. a) The bicyclist's velocity is the greatest at time $t = 0$. The tangent line at $t = 0$ has the greatest slope.

b) The bicyclist's acceleration is negative, since the slopes of the tangent line are decreasing with time.

54. a) The plane's velocity is greater at $t = 20$ seconds. We know this because the slope of the tangent line is greater at $t = 20$ then it is at $t = 6$.

b) The plane's acceleration is positive, since the velocity (slope of the tangent lines) is increasing over time.

55. a) $f'(t) = 0$ indicates that the rate of change is zero so the graph will be horizontal. The graph appears to be horizontal over the interval $7 < t < 11$ or $(7, 11)$.

b) $f''(t) = 0$ indicates where the rate of change of the graph is changing at a constant rate. This appears to be the intervals $2 < t < 4$ or $(2, 4)$; $7 < t < 11$ or $(7, 11)$; $13 < t < 15$ or $(13, 15)$.

c) $f''(t) > 0$ indicates that the rate of change of the graph is increasing. This occurs over the intervals $0 < t < 2$ or $(0, 2)$; $11 < t < 13$ or $(11, 13)$.

d) $f''(t) < 0$ indicates that the rate of change of the graph is decreasing. This occurs over the interval $4 < t < 7$ or $(4, 7)$.

d) Answers will vary. "Sales are increasing" mean that the rate of change of sales are positive, that is the graph is positively sloped. "The rate of sales is increasing" means that sales are increasing and increasing faster and faster, that is the graph is positively sloped and concave up.

56. a) The vehicle is accelerating where the graph is increasing at an increasing rate. This occurs over the intervals $0 < t < 2$ or $(0,2)$; $8 < t < 9$ or $(8,9)$.

b) The vehicle is decelerating where the graph is increasing at a decreasing rate. This occurs over the intervals $4 < t < 5$ or $(4,5)$; $11 < t < 13$ or $(11,13)$.

c) The vehicle is maintaining a constant velocity when the graph is increasing at a constant rate. This occurs on the intervals. $2 < t < 4$ or $(2,4)$; $5 < t < 8$ or $(5,8)$; $9 < t < 11$ or $(9,11)$; $13 < t < 18$ or $(13,18)$.

57. $S(t) = 2t^3 - 40t^2 + 220t + 160$

a) $S'(t) = 6t^2 - 80t + 220$

When $t = 1$,

$S'(1) = 6(1)^2 - 80(1) + 220 = 146$

After 1 month, sales are increasing at 146 thousand (146,000) dollars per month.

When $t = 2$,

$S'(2) = 6(2)^2 - 80(2) + 220 = 84$

After 2 month, sales are increasing at 84 thousand (84,000) dollars per month.

When $t = 4$,

$S'(4) = 6(4)^2 - 80(4) + 220 = -4$

After 4 months, sales are changing at a rate of -4 thousand (-4000) dollars per month.

b) $S''(t) = 12t - 80$

When $t = 1$, $S''(1) = 12(1) - 80 = -68$

After 1 month, the rate of change of sales are changing at a rate of -68 thousand $(-68,000)$ dollars per month squared.

When $t = 2$,

$S''(2) = 12(2) - 80 = -56$

After 2 months, the rate of change of sales are changing at a rate of -56 thousand $(-56,000)$ dollars per month squared.

When $t = 4$, $S''(4) = 12(4) - 80 = -32$

After 4 months, the rate of change of sales are changing at a rate of -32 thousand $(-32,000)$ dollars per month squared.

c) Answers will vary. The first derivative found in part (a) determined the rate at which sales were changing *t* months after the product was marketed. We saw that for the first 2 months, sales were increasing. However, in the 4th month, sales had started to decrease. The second derivative found in part (b) determined how fast the rate of change was changing. We saw that the rate at which sales were changing was negative. Which means that in the first two months when sales were increasing, they were doing so at a decreasing rate.

58. $N(t) = 2t^3 - 3t^2 + 2t$

a) $N'(t) = 6t^2 - 6t + 2$

When $t = 1$, $N'(1) = 6(1)^2 - 6(1) + 2 = 2$

After 1 day, the number of items sold was increasing by 2 items per day.

When $t = 2$, $N'(2) = 6(2)^2 - 6(2) + 2 = 14$

After 2 days, the number of items sold was increasing by 14 items per day.

When $t = 4$, $N'(4) = 6(4)^2 - 6(4) + 2 = 74$

After 4 days, the number of items sold was increasing by 74 items per day.

b) $N''(t) = 12t - 6$

When $t = 1$, $N''(1) = 12(1) - 6 = 6$

After 1 day, the rate of change of the number of items sold was increasing by 6 items per day squared.

When $t = 2$, $N''(2) = 12(2) - 6 = 18$

After 2 days, the rate of change of the number of items sold was increasing by 18 items per day squared.

When $t = 4$, $N''(4) = 12(4) - 6 = 42$

After 4 days, the rate of change of the number of items sold was increasing by 42 items per day squared.

c) Answers will vary. The first derivative found in part (a) determined the rate at which items were sold t days after the new sales promotion was launched. The second derivative found in part (b) determined the rate at which the rates in part (a) where changing. The information tells us that after the new sales promotion was launched, the number of items sold were increasing at a increasing rate with respect to time.

59. a) $p(t)=\frac{2000t}{4t+75}$

First find the derivative of the population function. Using the quotient rule, we have:

$$p'(t)$$
$$=\frac{d}{dt}\left(\frac{2000t}{4t+75}\right)$$
$$=\frac{(4t+75)\frac{d}{dt}(2000t)-(2000t)\frac{d}{dt}(4t+75)}{(4t+75)^2}$$
$$=\frac{(4t+75)(2000)-(2000t)(4)}{(4t+75)^2}$$
$$=\frac{8000t+150,000-8000t}{(4t+75)^2}$$
$$=\frac{150,000}{(4t+75)^2}.$$

Next we substitute the appropriate values in for t.

$$p'(10)=\frac{150,000}{(4(10)+75)^2}=\frac{150,000}{13,225}\approx 11.34 .$$
$$p'(50)=\frac{150,000}{(4(50)+75)^2}=\frac{150,000}{75,625}\approx 1.98 .$$
$$p'(100)=\frac{150,000}{(4(100)+75)^2}=\frac{150,000}{225,625}\approx 0.665 .$$

b) First find the second derivative.

$$p''(t)=\frac{d}{dt}\left(\frac{150,000}{(4t+75)^2}\right)$$
$$=\frac{d}{dt}\left(150,000(4t+75)^{-2}\right)$$
$$=-300,000(4t+75)^{-3}\cdot 4$$
$$=-\frac{1,200,000}{(4t+75)^3}$$

Next we substitute values for t:

$$p''(10)=-\frac{1,200,000}{(4(10)+75)^3}=-\frac{1,200,000}{1,520,875}\approx -0.789 .$$
$$p''(50)=-\frac{1,200,000}{(4(50)+75)^3}=-\frac{1,200,000}{20,796,875}\approx -0.0577 .$$
$$p''(100)=-\frac{1,200,000}{(4(100)+75)^3}=-\frac{1,200,000}{107,171,875}\approx -0.0112 .$$

c) The population of deer is increasing but at a decreasing rate. The growth of the deer population will eventually find a stable population around 500 deer.

60. a) $p(t)=\frac{2.5t}{t^2+1}$

$$p'(t)$$
$$=\frac{d}{dt}\left(\frac{2.5t}{t^2+1}\right)$$
$$=\frac{(t^2+1)\frac{d}{dt}(2.5t)-(2.5t)\frac{d}{dt}(t^2+1)}{(t^2+1)^2}$$
$$=\frac{(2.5t^2+2.5)-(5t^2)}{(t^2+1)^2}$$
$$=\frac{-2.5t^2+2.5}{(t^2+1)^2}.$$

The solution is continued on the next page.

Next we substitute the appropriate values in for t into the derivative found on the previous page.

$$p'(0.5)=\frac{-2.5(0.5)^2+2.5}{\left((0.5)^2+1\right)^2}$$
$$=1.2\ .$$
$$p'(1)=\frac{-2.5(1)^2+2.5}{\left((1)^2+1\right)^2}$$
$$=0\ .$$
$$p'(5)=\frac{-2.5(5)^2+2.5}{\left((5)^2+1\right)^2}$$
$$\approx -0.888\ .$$
$$p'(30)=\frac{-2.5(30)^2+2.5}{\left((30)^2+1\right)^2}$$
$$\approx -0.0028\ .$$

b) First find the second derivative.

$$p''(t)=\frac{d}{dt}\left(\frac{-2.5t^2+2.5}{\left(t^2+1\right)^2}\right)$$
$$=\frac{5t^3-15t}{\left(t^2+1\right)^3}$$

Substitute the appropriate values in for t.

$$p''(0.5)=\frac{5(0.5)^3-15(0.5)}{\left((0.5)^2+1\right)^3}=-3.52\ .$$
$$p''(1)=\frac{5(1)^3-15(1)}{\left((1)^2+1\right)^3}=-1.25\ .$$
$$p''(5)=\frac{5(5)^3-15(5)}{\left((5)^2+1\right)^3}\approx 0.031\ .$$
$$p''(30)=\frac{5(30)^3-15(30)}{\left((30)^2+1\right)^3}\approx 0.00018\ .$$

c) The medication initially increases, however it quickly begins to decrease at an increasing rate. The amount of medicine in the bloodstream will eventually be negligible.

61. $y=\frac{1}{(1-x)}=(1-x)^{-1}$

$$y'=-1(1-x)^{-1-1}(-1)=1(1-x)^{-2}$$
$$y''=-2(1-x)^{-2-1}(-1)=2(1-x)^{-3}$$
$$y'''=2(-3)(1-x)^{-3-1}(-1)=6(1-x)^{-4}$$

Therefore,

$$y'''=\frac{6}{(1-x)^4}$$

62. $y=\frac{1}{\sqrt{2x+1}}=(2x+1)^{-1/2}$

$$y'=\frac{-1}{2}(2x+1)^{-1/2-1}(2)$$
$$=-(2x+1)^{-3/2}$$
$$y''=-\frac{-3}{2}(2x+1)^{-3/2-1}(2)$$
$$=3(2x+1)^{-5/2}$$
$$y'''=3\left(\frac{-5}{2}\right)(2x+1)^{-5/2-1}(2)$$
$$=-15(2x+1)^{-7/2}$$
$$=-\frac{15}{(2x+1)^{7/2}}$$

63. $y=\frac{\sqrt{x}+1}{\sqrt{x}-1}=\frac{x^{1/2}+1}{x^{1/2}-1}$

$$y'=\frac{\left(x^{1/2}-1\right)\left(\frac{1}{2}x^{-1/2}\right)-\left(x^{1/2}+1\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}-1\right)^2}$$
$$=\frac{\frac{1}{2}-\frac{1}{2}x^{-1/2}-\frac{1}{2}-\frac{1}{2}x^{-1/2}}{\left(x^{1/2}-1\right)^2}$$
$$=\frac{-x^{-1/2}}{\left(x^{1/2}-1\right)^2}$$

We find the second derivative on the next page.

Using the information from the previous page, we find the second derivative.

$$y''=\frac{\left(x^{1/2}-1\right)^2\left(\frac{1}{2}x^{-3/2}\right)-\left(-x^{-1/2}\right)\left[2\left(x^{1/2}-1\right)\frac{1}{2}x^{-1/2}\right]}{\left(\left(x^{1/2}-1\right)^2\right)^2}$$

$$=\frac{\frac{1}{2}x^{-3/2}\left(x^{1/2}-1\right)^2+x^{-1}\left(x^{1/2}-1\right)}{\left(x^{1/2}-1\right)^4}$$

$$=\frac{\left(x^{1/2}-1\right)\left(\frac{1}{2}x^{-3/2}\left(x^{1/2}-1\right)+x^{-1}\right)}{\left(x^{1/2}-1\right)^4}$$

$$=\frac{\frac{1}{2}x^{-1}-\frac{1}{2}x^{-3/2}+x^{-1}}{\left(x^{1/2}-1\right)^3}$$

$$=\frac{\left(\frac{3}{2}x^{-1}-\frac{1}{2}x^{-3/2}\right)}{\left(x^{1/2}-1\right)^3}$$

$$=\frac{\frac{3}{2x}-\frac{1}{2x^{3/2}}}{\left(x^{1/2}-1\right)^3}$$

$$=\frac{\frac{3x^{1/2}}{2x^{3/2}}-\frac{1}{2x^{3/2}}}{\left(x^{1/2}-1\right)^3}$$

$$=\frac{3x^{1/2}-1}{2x^{3/2}\left(x^{1/2}-1\right)^3}$$

64. $y=\frac{x}{\sqrt{x-1}}=\frac{x}{(x-1)^{1/2}}$

$$y'=\frac{(x-1)^{1/2}(1)-x\left(\frac{1}{2}(x-1)^{-1/2}(1)\right)}{\left((x-1)^{1/2}\right)^2}$$

$$=\frac{(x-1)^{1/2}-\frac{x}{2(x-1)^{1/2}}}{x-1}$$

$$=\frac{\frac{2(x-1)}{2(x-1)^{1/2}}-\frac{x}{2(x-1)^{1/2}}}{x-1}$$

The derivative is simplified at the top of the next column.

Simplifying, we have:

$$y'=\frac{\frac{x-2}{2(x-1)^{1/2}}}{x-1}$$

$$=\frac{x-2}{2(x-1)^{3/2}}$$

$$y''=\frac{1}{2}\cdot\frac{(x-1)^{3/2}(1)-(x-2)\left(\frac{3}{2}(x-1)^{1/2}(1)\right)}{\left((x-1)^{3/2}\right)^2}$$

$$=\frac{1}{2}\cdot\frac{(x-1)^{1/2}\left[(x-1)-\frac{3}{2}(x-2)\right]}{(x-1)^3}$$

$$=\frac{1}{2}\cdot\frac{x-1-\frac{3}{2}x+3}{(x-1)^{5/2}}$$

$$=\frac{1}{2}\cdot\frac{2-\frac{1}{2}x}{(x-1)^{5/2}}$$

$$=\frac{1-\frac{1}{4}x}{(x-1)^{5/2}}$$

65. $y=x^k$

$$\frac{dy}{dx}=kx^{k-1}$$

$$\frac{d^2y}{dx^2}=k(k-1)x^{k-2}$$

$$\frac{d^3y}{dx^3}=k(k-1)(k-2)x^{k-3}$$

$$\frac{d^4y}{dx^4}=k(k-1)(k-2)(k-3)x^{k-4}$$

$$\frac{d^5y}{dx^5}=k(k-1)(k-2)(k-3)(k-4)x^{k-5}$$

66. $y=ax^3+bx^2+cx+d$

$$\frac{dy}{dx}=3ax^2+2bx+c$$

$$\frac{d^2y}{dx^2}=6ax+2b$$

$$\frac{d^3y}{dx^3}=6a$$

67. $f(x)=\dfrac{x-1}{x+2}$

$$f'(x)=\frac{(x+2)(1)-(x-1)(1)}{(x+2)^2}$$
$$=\frac{3}{(x+2)^2}$$

Notice, $f'(x)=\dfrac{3}{(x+2)^2}=3(x+2)^{-2}$ so,

$$f''(x)=3(-2)(x+2)^{-2-1}\cdot(1)$$
$$=-6(x+2)^{-3}$$
$$=-\frac{6}{(x+2)^3}$$

Notice, $f''(x)=-\dfrac{6}{(x+2)^3}=-6(x+2)^{-3}$ so,

$$f'''(x)=-6(-3)(x+2)^{-3-1}(1)$$
$$=18(x+2)^{-4}$$
$$=\frac{18}{(x+2)^4}$$

Notice, $f'''(x)=\dfrac{18}{(x+2)^4}=18(x+2)^{-4}$ so,

$$f^{(4)}(x)=18(-4)(x+2)^{-4-1}(1)$$
$$=-72(x+2)^{-5}$$
$$=-\frac{72}{(x+2)^5}$$

68. $f(x)=\dfrac{x+3}{x-2}$

$$f'(x)=\frac{(x-2)(1)-(x+3)(1)}{(x-2)^2}$$
$$=\frac{-5}{(x-2)^2}=-5(x-2)^{-2}$$
$$f''(x)=-5(-2)(x-2)^{-3}(1)$$
$$=10(x-2)^{-3}$$
$$=\frac{10}{(x-2)^3}$$
$$f'''(x)=10(-3)(x-2)^{-3-1}(1)$$
$$=-30(x-2)^{-4}$$
$$=-\frac{30}{(x-2)^4}$$
$$f^{(4)}(x)=-30(-4)(x-2)^{-4-1}(1)$$
$$=120(x-2)^{-5}$$
$$=\frac{120}{(x-2)^5}$$

69. Consider the function $s(t)=4.905t^2$. We find the first and second derivatives.

$s'(t)=4.905\left(2t^{2-1}\right)=9.81t$

$s''(t)=9.81$

The second derivative, $s''(t)=9.81$, represents acceleration due to gravity on Earth which is 9.81 meters per second squared.

70.
a) $s(2)=0.81(2)^2\approx 3.24$
The object has fallen 3.24 meters after 2 seconds.

b) To determine the speed, we find the first derivative of the position function, which is $s'(t)=1.62t$.
Therefore,
$s'(2)=1.62(2)=3.24$.
The object is traveling at 3.24 meters per second after 2 seconds.

c) To determine the acceleration, we find the second derivative of the position function, which is:
$s''(t)=1.62$.
The object is accelerating at 1.62 meters per second squared.

d) The second derivative represents the acceleration due to gravity on the moon. It is a constant $1.62\dfrac{m}{\sec^2}$.

71. Answers will vary. 2 seconds is impossible, the top high jumpers clear about 2 meters in height. So it is possible for a human to have a "hang time" of 1 second, maybe even 1.5 seconds.

72. First we must find out how long it took to fall 28 ft. Solving the equation

$$s(t) = 28$$
$$16t^2 = 28$$
$$t^2 = 1.75$$
$$t = \pm 1.3229$$

We find out it took about 1.3229 seconds to fall to the ramp.
Next, we find the velocity function which is the first derivative of the position function. The first derivative is

$$s'(t) = 32t.$$

Substituting 1.3229 in for t, we have

$$s'(1.32) = 32(1.3229) = 42.33$$

Danny Way was traveling around 42.33 feet per second when he touched down on the ramp.

73. a) Graph I matches the description.
b) Graph IV matches the description.
c) Graph II matches the description.
d) Graph III matches the description.

74. ✎ Answers will vary.

75. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 5} \frac{x^2-25}{2x-10} = \frac{(5)^2-25}{2(5)-10} = \frac{0}{0}.$$

This is an indeterminate form.
We now apply L'Hôpital's Rule to find the limit. By taking the derivative of the numerator and denominator separately we have:

$$\lim_{x\to 5} \frac{x^2-25}{2x-10} = \lim_{x\to 5}\left(\frac{\frac{d}{dx}(x^2-25)}{\frac{d}{dx}(2x-10)}\right) = \lim_{x\to 5}\left(\frac{2x}{2}\right) = \lim_{x\to 5}(x) = 5. \quad \text{Substitution}$$

76. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to -2} \frac{x^2-4}{x+2} = \frac{(-2)^2-4}{(-2)+2} = \frac{0}{0}.$$

This is an indeterminate form.
We now apply L'Hôpital's Rule to find the limit.

$$\lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2}\left(\frac{2x}{1}\right) = -4.$$

77. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 1} \frac{x^3+2x-3}{x^2-1} = \frac{(1)^3+2(1)-3}{(1)^2-1} = \frac{0}{0}.$$

This is an indeterminate form.
We now apply L'Hôpital's Rule to find the limit. By taking the derivative of the numerator and denominator separately we have:

$$\lim_{x\to 1} \frac{x^3+2x-3}{x^2-1} = \lim_{x\to 1}\left(\frac{\frac{d}{dx}(x^3+2x-3)}{\frac{d}{dx}(x^2-1)}\right) = \lim_{x\to 1}\left(\frac{3x^2+2}{2x}\right) = \frac{3(1)^2+2}{2(1)} \quad \text{Substitution} \quad = \frac{5}{2}.$$

78. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 3} \frac{x^3-x-24}{x^2-9} = \frac{(3)^3-(3)-24}{(3)^2-9} = \frac{0}{0}.$$

This is an indeterminate form.
We now apply L'Hôpital's Rule to find the limit.

$$\lim_{x\to 3} \frac{x^3-x-24}{x^2-9} = \lim_{x\to 3}\left(\frac{3x^2-1}{2x}\right) = \frac{13}{3}.$$

79. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to -3} \frac{x^2-9}{x+3} = \frac{(-3)^2-9}{(-3)+3} = \frac{0}{0}.$$

This is an indeterminate form.
The solution is continued on the next page.

We now apply L'Hôpital's Rule to find the limit. By taking the derivative of the numerator and denominator on the previous page separately we have:

$$\lim_{x\to -3}\frac{x^2-9}{x+3}=\lim_{x\to -3}\left(\frac{\frac{d}{dx}\left(x^2-9\right)}{\frac{d}{dx}\left(x+3\right)}\right)$$
$$=\lim_{x\to -3}\left(\frac{2x}{1}\right)$$
$$=\lim_{x\to -3}\left(2x\right)$$
$$=2\left(-3\right) \qquad \text{Substitution}$$
$$=-6.$$

80. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to -4}\frac{x^2+x-12}{x+4}=\frac{\left(-4\right)^2-\left(-4\right)-12}{\left(-4\right)+4}=\frac{0}{0}.$$

This is an indeterminate form.
We now apply L'Hôpital's Rule to find the limit.

$$\lim_{x\to -4}\frac{x^2+x-12}{x+4}=\lim_{x\to -4}\left(\frac{2x+1}{1}\right)$$
$$=-7.$$

81. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 2}\frac{x^3+5x-18}{2x^2-8}=\frac{\left(2\right)^3+5\left(2\right)-18}{2\left(2\right)^2-8}$$
$$=\frac{0}{0}.$$

This is an indeterminate form.
We now apply L'Hôpital's Rule to find the limit. By taking the derivative of the numerator and denominator separately we have:

$$\lim_{x\to 2}\frac{x^3+5x-18}{2x^2-8}=\lim_{x\to 2}\left(\frac{\frac{d}{dx}\left(x^3+5x-18\right)}{\frac{d}{dx}\left(2x^2-8\right)}\right)$$
$$=\lim_{x\to 2}\left(\frac{3x^2+5}{4x}\right)$$
$$=\frac{3\left(2\right)^2+5}{4\left(2\right)} \qquad \text{Substitution}$$
$$=\frac{17}{8}.$$

82. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 10}\frac{x^2+x-110}{x-10}=\frac{\left(10\right)^2+\left(10\right)-110}{\left(10\right)-10}=\frac{0}{0}.$$

This is an indeterminate form.
We now apply L'Hôpital's Rule to find the limit.

$$\lim_{x\to 10}\frac{x^2+x-110}{x-10}=\lim_{x\to 10}\left(\frac{2x+1}{1}\right)$$
$$=21.$$

83. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to\infty}\frac{4x^2+x-3}{2x^2+1}=\frac{\infty}{\infty}.$$

This is an indeterminate form.
We now apply L'Hôpital's Rule to find the limit. By taking the derivative of the numerator and denominator separately we have:

$$\lim_{x\to\infty}\frac{4x^2+x-3}{2x^2+1}=\lim_{x\to\infty}\left(\frac{\frac{d}{dx}\left(4x^2+x-3\right)}{\frac{d}{dx}\left(2x^2+1\right)}\right)$$
$$=\lim_{x\to\infty}\left(\frac{8x+1}{4x}\right)$$
$$=\frac{\infty}{\infty} \qquad \text{Substitution}$$

This is still indeterminate so we apply L'Hôpital's Rule again.

$$\lim_{x\to\infty}\frac{4x^2+x-3}{2x^2+1}=\lim_{x\to\infty}\left(\frac{8x+1}{4x}\right)$$
$$=\lim_{x\to\infty}\left(\frac{\frac{d}{dx}\left(8x+1\right)}{\frac{d}{dx}\left(4x\right)}\right)$$
$$=\lim_{x\to\infty}\left(\frac{8}{4}\right)$$
$$=2.$$

84. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to\infty}\frac{3x^3+x+11}{6x^3+x+2}=\frac{\infty}{\infty}.$$

This is an indeterminate form.
We now apply L'Hôpital's Rule to find the limit.

$$\lim_{x\to\infty}\frac{3x^3+x+11}{6x^3+x+2}=\lim_{x\to\infty}\left(\frac{9x^2+1}{18x^2+1}\right)=\frac{\infty}{\infty}.$$

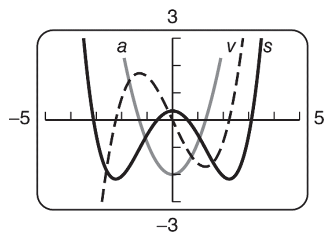
This is still an indeterminate form.

The solution is continued on the next page.

We now apply L'Hôpital's Rule two more times.

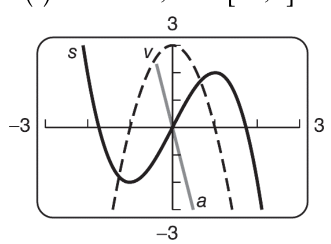
$$\lim_{x\to\infty}\frac{3x^3+x+11}{6x^3+x+2}=\lim_{x\to\infty}\left(\frac{9x^2+1}{18x^2+1}\right)$$
$$=\lim_{x\to\infty}\left(\frac{18x}{36x}\right)$$
$$=\lim_{x\to\infty}\left(\frac{18}{36}\right)$$
$$=\frac{1}{2}.$$

85. $s(t)=0.1t^4-t^2+0.4; \qquad [-5,5]$



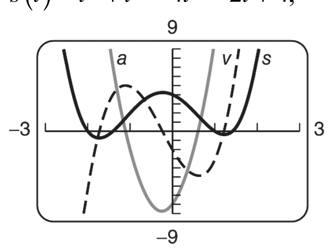
From the graph we see that $v(t)$ switches at $t=-1.29$ and $t=1.29$.

86. $s(t)=-t^3+3t; \qquad [-3,3]$



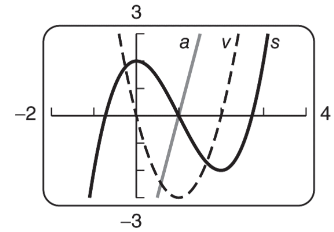
From the graph we see that $v(t)$ switches at $t=0$.

87. $s(t)=t^4+t^3-4t^2-2t+4; \qquad [-3,3]$



From the graph we see that $v(t)$ switches at $t=0.604$ and $t=-1.104$.

88. $s(t)=t^3-3t^2+2; \qquad [-2,4]$



From the graph we see that $v(t)$ switches at $t=1$.